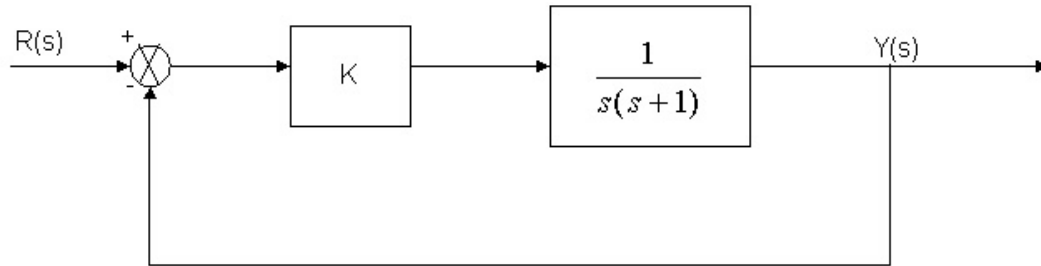


HANDOUT E.19 - EXAMPLES ON FEEDBACK CONTROL SYSTEMS

Example 1

Consider the system shown below.



The open loop transfer function is given by

$$G(s) = \frac{K}{s(s+1)}.$$

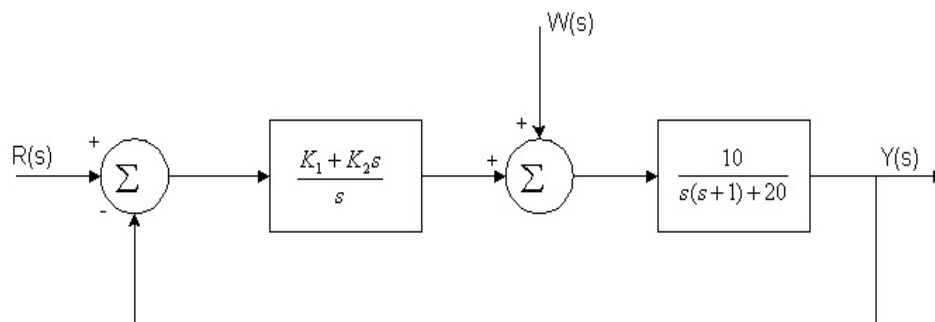
The closed loop transfer function is

$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{\frac{K}{s(s+1)}}{1 + \frac{K}{s(s+1)}} = \frac{K}{s(s+1) + K} = \frac{K}{s^2 + s + K}.$$

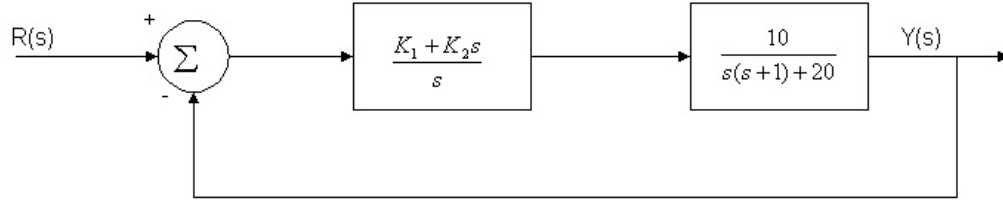
Example 2

Consider the system shown.

- Determine the transfer function from 'r' to 'y'.
- Determine the transfer function from 'w' to 'y'.



To obtain the transfer function between 'r' to 'y', assume 'w' to be equal to zero. Therefore the block diagram reduces to,



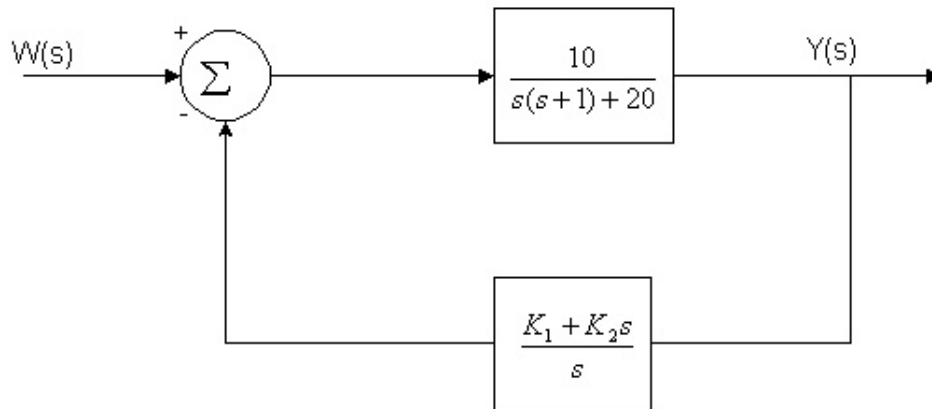
Hence the open loop transfer function is

$$G(s) = \frac{(K_1 + K_2s)}{s} \cdot \frac{10}{s(s+1)+20} = \frac{10(K_1 + K_2s)}{s(s^2 + s + 20)}.$$

The closed loop transfer function is given by

$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{\frac{10(K_1 + K_2s)}{s(s^2 + s + 20)}}{1 + \frac{10(K_1 + K_2s)}{s(s^2 + s + 20)}} = \frac{10(K_1 + K_2s)}{s^3 + s^2 + s(20 + 10K_2) + 10K_1}.$$

To obtain the transfer function between 'w' and 'y', assume 'r' to be equal to zero. Hence the block diagram reduces to,



For the above block diagram the open loop transfer function is

$$G(s) = \frac{10}{s(s+1)+20} = \frac{10}{s^2 + s + 20}$$

and the feedback transfer function is given by

$$H(s) = \frac{K_1 + K_2s}{s}.$$

Therefore the transfer function from 'w' to 'y' is given by

$$\frac{Y(s)}{W(s)} = \frac{G(s)}{1 + G(s)H(s)} = \frac{\frac{10}{(s^2 + s + 20)}}{1 + \frac{10}{(s^2 + s + 20)} \cdot \frac{K_1 + K_2 s}{s}} = \frac{10s}{(s^3 + s^2 + s(20 + 10K_2) + 10K_1)}.$$

Example 3

A unity feedback system has the open loop transfer function

$$G(s) = \frac{s+1}{s^3 + s^2}.$$

Find the error constants K_p , K_v and K_a for the system.

The closed loop transfer function is given as

$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)} = \frac{\frac{s+1}{s^3 + s^2}}{1 + \frac{s+1}{s^3 + s^2}} = \frac{s+1}{s^3 + s^2 + s + 1}.$$

The transfer function for the error signal is given by

$$E(s) = R(s) - Y(s) = R(s) - \frac{s+1}{s^3 + s^2 + s + 1} R(s) = \frac{s^3 + s^2}{s^3 + s^2 + s + 1} R(s).$$

Therefore the steady state error for the system can be obtained by applying the final value theorem as

$$e_{ss} = \lim_{s \rightarrow 0} sE(s).$$

Therefore,

$$e_{ss} = \lim_{s \rightarrow 0} s \frac{s^3 + s^2}{s^3 + s^2 + s + 1} R(s).$$

For a unit step input, we have $R(s) = \frac{1}{s}$, therefore the steady state error is

$$e_{ss} = \lim_{s \rightarrow 0} s \cdot \frac{s^3 + s^2}{s^3 + s^2 + s + 1} \cdot \frac{1}{s} = \lim_{s \rightarrow 0} \frac{s^3 + s^2}{s^3 + s^2 + s + 1} = 0 = \frac{1}{1 + K_p}.$$

Therefore the value of K_p is given as

$$K_p \rightarrow \infty.$$

For a ramp input, we have $R(s) = 1/s^2$, therefore the steady state error is

$$e_{ss} = \lim_{s \rightarrow 0} s \cdot \frac{s^3 + s^2}{s^3 + s^2 + s + 1} \cdot \frac{1}{s^2} = \lim_{s \rightarrow 0} \frac{s^2 + s}{s^3 + s^2 + s + 1} = 0 = \frac{1}{K_v}.$$

Hence the value of K_v is given as

$$K_v \rightarrow \infty.$$

For a parabolic input, we have $R(s) = 1/s^3$, therefore the steady state error is

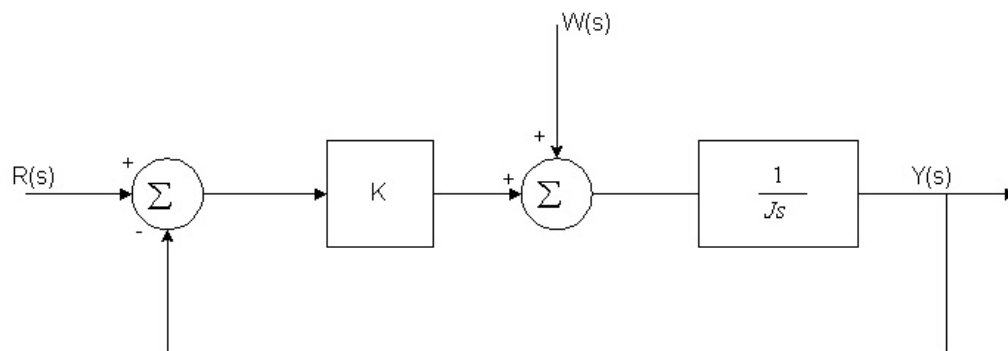
$$e_{ss} = \lim_{s \rightarrow 0} s \cdot \frac{s^3 + s^2}{s^3 + s^2 + s + 1} \cdot \frac{1}{s^3} = \lim_{s \rightarrow 0} \frac{s + 1}{s^3 + s^2 + s + 1} = 1 = \frac{1}{K_a}.$$

Therefore,

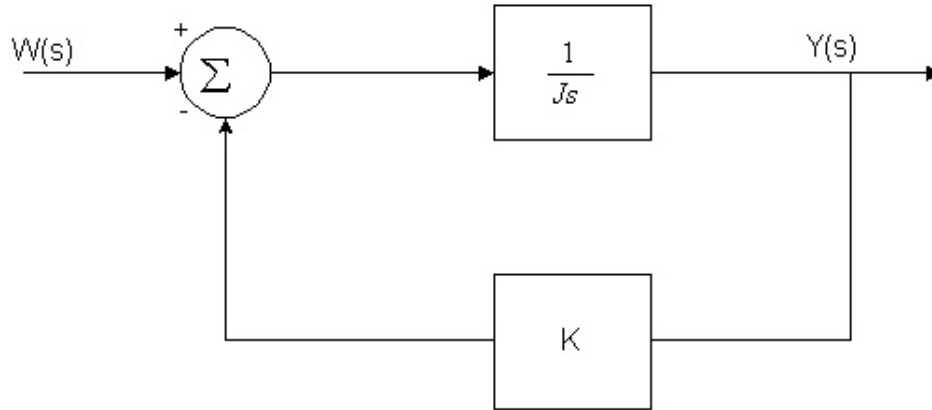
$$K_a = 1.$$

Example 4

The block diagram shown below shows a control system in which the output member of the system is subject to a disturbance. In the absence of a disturbance, the output is equal to the reference. Investigate the response of this system to a unit step disturbance.



Since we are interested in the response of the system to a unit step disturbance, assume the reference input to be equal to zero. Therefore the block diagram reduces to



The open loop transfer function is given by

$G(s) = \frac{1}{Js}$, and the feedback transfer function is given as

$H(s) = K$.

Therefore the transfer function between the disturbance and the output is given as

$$\frac{Y(s)}{W(s)} = \frac{G(s)}{1 + G(s)H(s)} = \frac{\frac{1}{Js}}{1 + \frac{K}{Js}} = \frac{1}{Js + K}.$$

For a unit step disturbance, the output reduces to

$$Y(s) = \frac{1}{Js + K} \cdot \frac{1}{s} = \frac{1}{s(Js + K)}.$$

The steady state value of the output can be obtained by applying the final value theorem as

$$y_{ss} = \lim_{s \rightarrow 0} sY(s) = \lim_{s \rightarrow 0} s \cdot \frac{1}{s(Js + K)} = \lim_{s \rightarrow 0} \frac{1}{Js + K} = \frac{1}{K}.$$

Therefore the system is incapable of rejecting the disturbance completely as there is a steady state offset.

Example 5

Consider a nonunity feedback system, whose open loop transfer function is given by

$$G(s) = \frac{1}{s(1 + Ts)},$$

where in the feedback loop transfer function is given as

$H(s) = h$, with $h > 0$. Determine the system type with respect to the reference input.

The closed loop transfer function is given as

$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} = \frac{\frac{1}{s(1+Ts)}}{1 + \frac{h}{s(1+Ts)}} = \frac{1}{Ts^2 + s + h}.$$

The transfer function for the error signal is given by

$$E(s) = R(s) - Y(s) = \left(1 - \frac{1}{Ts^2 + s + h}\right)R(s) = \frac{Ts^2 + s + h - 1}{Ts^2 + s + h}R(s).$$

The steady state error is obtained as

$$e_{ss} = \lim_{s \rightarrow 0} s \cdot E(s) = \lim_{s \rightarrow 0} s \cdot \frac{Ts^2 + s + h - 1}{Ts^2 + s + h} R(s).$$

For a step reference input, the error is

$$e_{ss} = \lim_{s \rightarrow 0} s \cdot \frac{Ts^2 + s + h - 1}{Ts^2 + s + h} \cdot \frac{1}{s} = \lim_{s \rightarrow 0} \frac{Ts^2 + s + h - 1}{Ts^2 + s + h} = \frac{h - 1}{h}.$$

If the system is not unity feedback then, $h \neq 1$, therefore the system is of Type 0. If the system is unity feedback, i.e., $h = 1$, then the system is of Type 1 as the steady state error for the step reference input is zero.

Assignment

1) Consider a unity feedback system whose open loop transfer function is given by

$$G(s) = \frac{(s+1)}{s(s^2 + 2s + 5)}.$$

Determine the closed loop transfer function of the system.

2) Problem 4.32a from the textbook “Feedback control of dynamic systems”, by Franklin et.al.

Recommended Reading

“Feedback Control of Dynamic Systems” 4th Edition, by Gene F. Franklin et.al – pp 201 – 215, 230 – 242.

Recommended Assignment

“Feedback Control of Dynamic Systems” 4th Edition, by Gene F. Franklin et.al – problems 4.22, 4.28, 4.33, 4.35, 4.37.