HANDOUT E.22 - EXAMPLES ON STABILITY ANALYSIS

Example 1

Determine the stability of the system whose characteristics equation given by

$$a(s) = s^6 + 4s^5 + 3s^4 + 2s^3 + s^2 + 4s + 4.$$

The above polynomial satisfies the necessary condition for stability since all the coefficients are positive and nonzero. Writing the Routh array, we have

$$
\begin{array}{c|cccc}
s^6 & 1 & 3 & 1 & 4 \\
s^5 & 4 & 2 & 4 & 0 \\
s^4 & \frac{5}{2} = \frac{4.3 - 1.2}{4} & 0 = \frac{4.1 - 4.1}{4} & 4 = \frac{4.4 - 1.0}{4} & \\
s^3 & 2 = \frac{\frac{5}{2}.2 - 4.0}{\frac{5}{2}} & -\frac{12}{5} = \frac{\frac{5}{2}.4 - 4.4}{\frac{5}{2}} & 0 & \\
s^2 & 3 = \frac{2.0 - \frac{5}{2}(-\frac{12}{5})}{2} & 4 = \frac{2.4 - \frac{5}{2}.0}{2} & & \\
s^1 & -\frac{76}{15} = \frac{3(-\frac{12}{5}) - 4.2}{3} & 0 & & \\
s^0 & 4 = \frac{-\frac{76}{15}.4 - 0}{-\frac{76}{15}} & & &
\end{array}
$$

We conclude that the system has roots in the right half plane, since the elements of the first column are not all positive. In fact there are two roots in the right half plane, since there are two sign changes. In other words two closed loop poles of the system lie in the right half plane and hence the system is unstable.

Example 2

Determine the stability of the following polynomial.

$$a(s) = s^5 + 5s^4 + 11s^3 + 23s^2 + 28s + 12.$$

Writing the Routh array, we have

s^5	1	11	28
s^4	5	23	12
s^3	6.4	25.6	0
s^2	3	12	
s^1	0	0	

Since the entire row is zero, we construct an auxiliary equation by taking the coefficients of the previous row, i.e.,

$$a_1(s) = 3s^2 + 12.$$

Differentiating the above equation with respect to 's', we get

$$\frac{da_1(s)}{ds} = 6s. \tag{1}$$

So the Routh array is continued by taking the coefficients of equation (1).

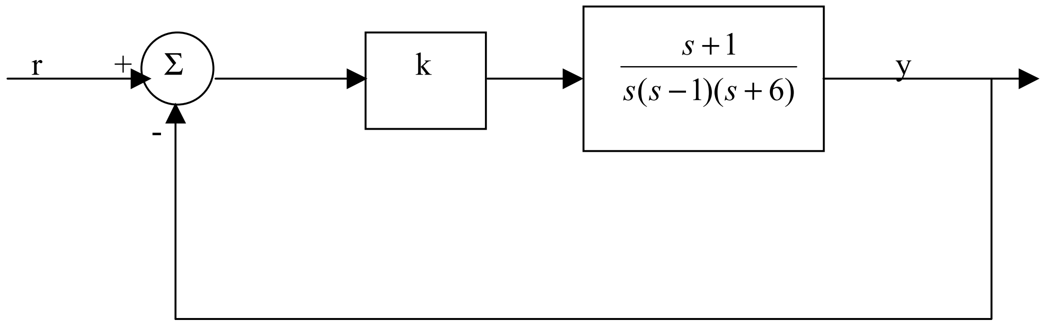
s^1	6	0
s^0	12	

Since there are no sign changes in the first column of the Routh array, there are no roots in the right half plane. However, since one entire row in the Routh array was zero, there are roots in the imaginary axis. The roots in the imaginary axis can be obtained by solving the auxiliary equation. Therefore,

$$3s^2 + 12 = 0,$$
$$\Rightarrow s^2 + 4 = 0,$$
$$\Rightarrow s = \pm j2.$$

Example 3

Consider the system shown below. The stability properties of the system are a function of the proportional feedback gain ‘k’. Determine the range of ‘k’ over which the system is asymptotically stable.



The characteristics equation for the system is given by

$$1+k\frac{s+1}{s(s-1)(s+6)}=0,$$
$$\Rightarrow s^3+5s^2+(k-6)s+k=0.$$

Therefore the corresponding Routh array is

s^3	1	k-6
s^2	5	k
s^1	$\frac{(4k-30)}{5}$	
s^0	k	

For the system to be stable, it is necessary that all the elements in the first column of the Routh array must be positive. Therefore,

$$\frac{4k-30}{5}>0 \quad \text{and} \quad k>0,$$
$$\Rightarrow k>7.5 \quad \text{and} \quad k>0,$$
$$\Rightarrow k>7.5$$

Example 4

The state-space representation of a system is given as

$$\dot{\mathbf{x}} = \begin{bmatrix} -7 & -12 \\ 1 & 0 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}u,$$
$$y = \begin{bmatrix} 1 & 2 \end{bmatrix}\mathbf{x}.$$

For the system to be stable, the poles of the system should lie in the left half plane. In other words all the real poles should be negative or the real parts of complex poles must be negative. The poles of the system are nothing but the eigenvalues of the '**A**' matrix of the system. The MATLAB code is shown below

```
a = [-7 -12; 1 0];
[v,d] = eig(a)
```

The result is

```
v =

   -0.9701    0.9487
    0.2425   -0.3162


d =

    -4     0
     0    -3
```

The diagonal elements of the matrix 'd' are eigenvalues of the system and columns of the matrix 'v' represent the corresponding eigenvectors.

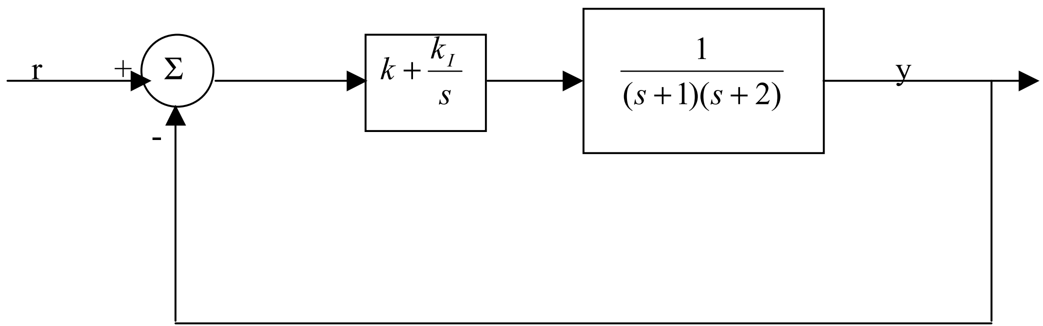
Note that, since all the eigenvalues are negative, the system is stable.

Assignment

1) Determine the stability of the system whose characteristics equation is given by

$$a(s) = s^5 + 3s^4 + 2s^3 + 6s^2 + 6s + 9.$$

2) Find the range of the controller gains (k, k_I) so that the PI feedback system is stable.



3) If the '**A**' matrix of the system is given by

$$\mathbf{A} = \begin{bmatrix} 0 & 4 & 0 & 0 \\ -1 & -4 & 0 & 0 \\ 5 & 7 & 1 & 15 \\ 0 & 0 & 3 & -3 \end{bmatrix}.$$

Determine the stability of the system?

Recommended Reading

"Feedback Control of Dynamic Systems" Fourth Edition, by Gene F. Franklin et.al – pp 157 - 166.

Recommended Assignment

"Feedback Control of Dynamic Systems" Fourth Edition, by Gene F. Franklin et.al – problems 3.39, 3.40, 3.41, 3.42.