HANDOUT E.26 - EXAMPLE HANDOUT ON ROOT LOCUS

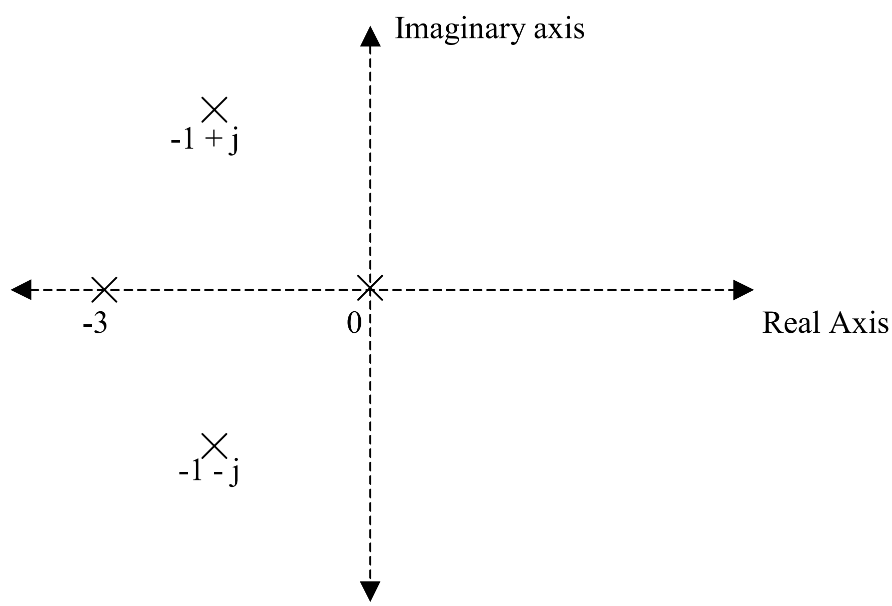
Example 1: A feedback control system has an open-loop transfer function

$$G(s)H(s) = \frac{K}{s(s+3)(s^2+2s+2)}$$

Find the root locus as K is varied from 0 to ∞.

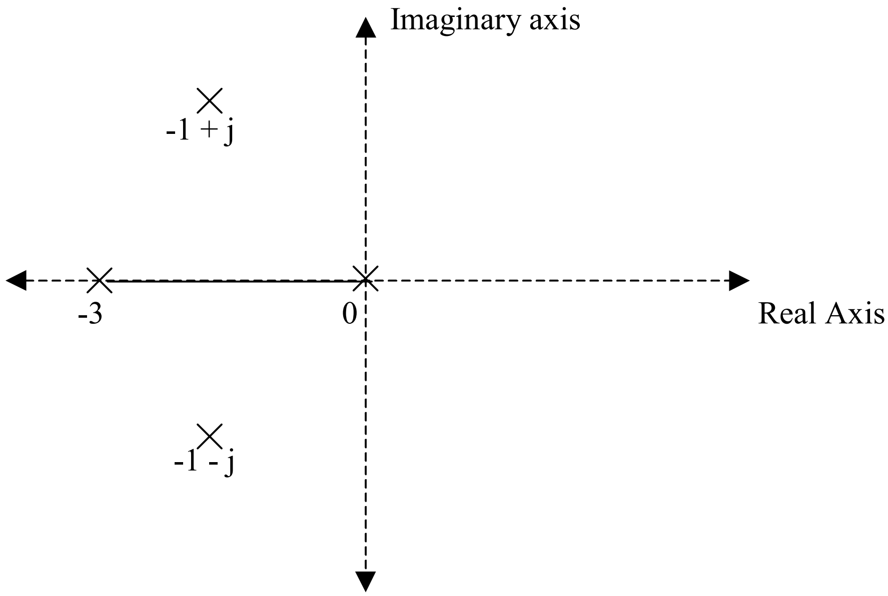
Sol:

Step 1: The root locus is symmetrical about the real axis. Mark the open-loop poles and zeros in the s-plane.



Step 2: Each branch of the root locus originates from the open-loop pole at K = 0, and terminates at K = ∞, either on an open-loop zero or infinity. Since the system given does not have any zero, all the branches terminate at infinity.

Step 3: Segments of the real axis having an odd number of real axis open-loop poles plus zeros to their right are parts of the root locus. Therefore representing the real axis root locus in the s-plane, we get



Step 4: The number of root locus branches equals the number of open-loop poles. Therefore in this case, the number of root locus branches is equal to 4.

Step 5: The root locus branches that tend to infinity, do so along the straight-line asymptotes making angles with real axis given by

$$\phi = \frac{180(2n+1)}{(Number\ of\ poles - Number\ of\ zeros)},$$

n = 0, 1,..., (number of poles – number of zeros)

Therefore the angle of the asymptotes for the system defined is given by

$$\phi = 45,\ 135,\ 225,\ 315.$$

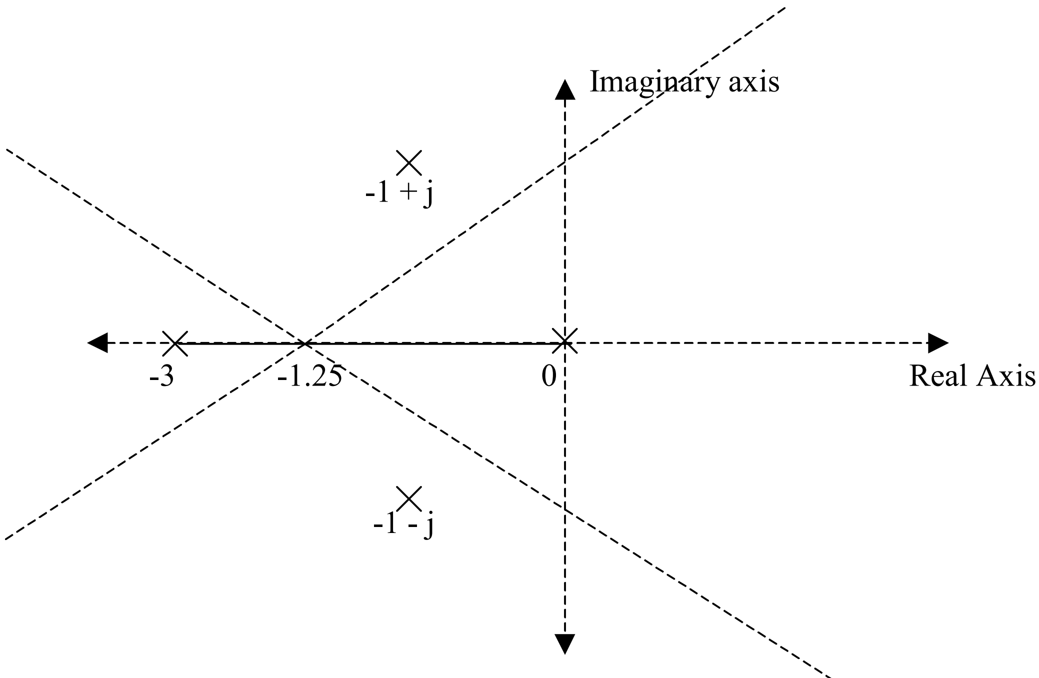
The point of intersection of the asymptotes with the real axis is given by

$$\alpha = \frac{\sum real\ parts\ of\ poles - \sum real\ parts\ of\ zeros}{(Number\ of\ poles - Number\ of\ zeros)}$$

Therefore the point of the intersection is given by

$$\alpha = \frac{(0-3-1-1)-(0)}{(4-0)} = -\frac{5}{4} = -1.25.$$

Representing the asymptotes in the s-plane, we have



Step 6: The breakaway points of the root locus are determined from the roots of the equation $\frac{dK}{ds} = 0.$

The characteristic equation of the system is given by

$$1 + G(s)H(s) = 0,$$
$$\Rightarrow 1 + \frac{K}{s(s+3)\left(s^2 + 2s + 2\right)} = 0,$$
$$\Rightarrow K = -s(s+3)\left(s^2 + 2s + 2\right)$$

$$\frac{dK}{ds} = -4\left(s^3 + 3.75s^2 + 4s + 1.5\right) = 0,$$
$$\Rightarrow s = -2.3,\ -0.725 \pm j0.365.$$

A breakaway point must occur at s = -2.3 as this part of the real axis is on the root locus and the two root locus branches starting at s = 0 and s = -3 are approaching each other.

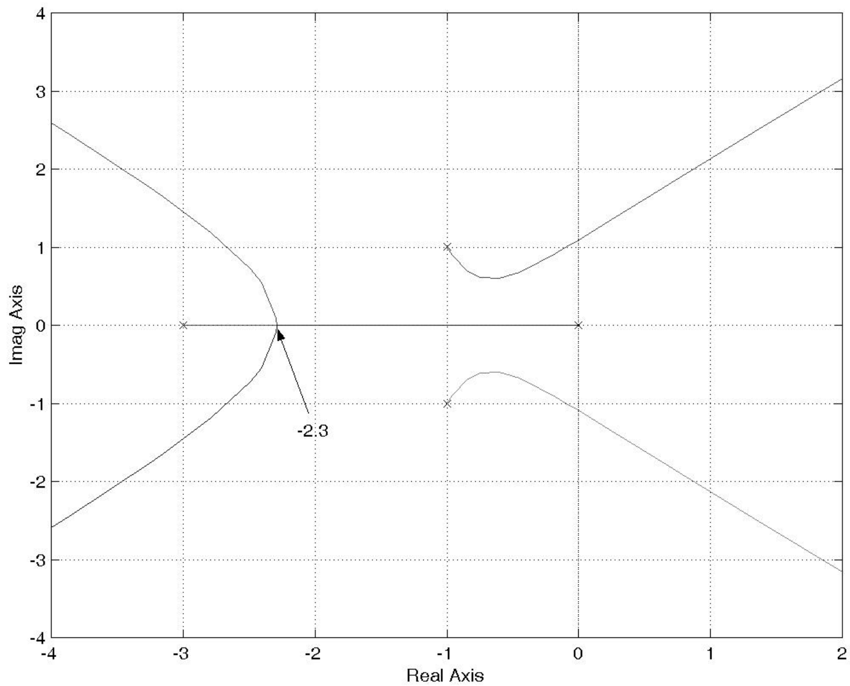
Step 7: The angle of departure from the open-loop pole is obtained by subtracting the sum of the angles subtended by the zero with the complex pole from the sum of the angles subtended by all the open loop poles and then adding the result to 180°. In other words,

$$\phi = \sum \angle all\ \ zeros - \sum \angle all\ \ poles + 180.$$

Note that, the angle is always measured in the counter clockwise direction.

In this case, the angle of departure from the open loop pole –1 + j is –71.6. Similarly the angle of departure from the other complex pole is also calculated.

The final root locus is then plotted as



To determine the value of K at which the root locus cuts the imaginary axis

The characteristic equation of the system is given by

$$s^4 + 5s^3 + 8s^2 + 6s + K = 0.$$

Building the Routh array, we get

s^4	1	8	K
s^3	5	6	
s^2	34/5	K	
s^1	$\frac{204/5 - 5K}{34/5}$		
s^0	K		

From the Routh array, it can be seen that for the system to be stable, we have

$$K > 0,$$
$$and$$
$$K < 8.16$$

Therefore the value of K at which the root locus cuts the imaginary axis is K = 8.16.

Once the value of K is known, the frequency at which the root locus cuts the imaginary axis can be obtained using the following relation,

$$|KG(s)| = 1.$$

Example 2: Using the guidelines shown in example 1, sketch the root locus for a unity feedback system around

$$G(s) = \frac{1}{s(s+c)}, \text{ as c varies from 0 to } \infty.$$

Sol: The characteristics equation of the system is given by

$$1 + G(s) = 0,$$
$$\Rightarrow 1 + \frac{1}{s(s+c)} = 0,$$
$$\Rightarrow s^2 + cs + 1 = 0.$$

Rearranging the terms in the characteristic equation, we get

$$1 + \frac{cs}{s^2+1} = 0.$$

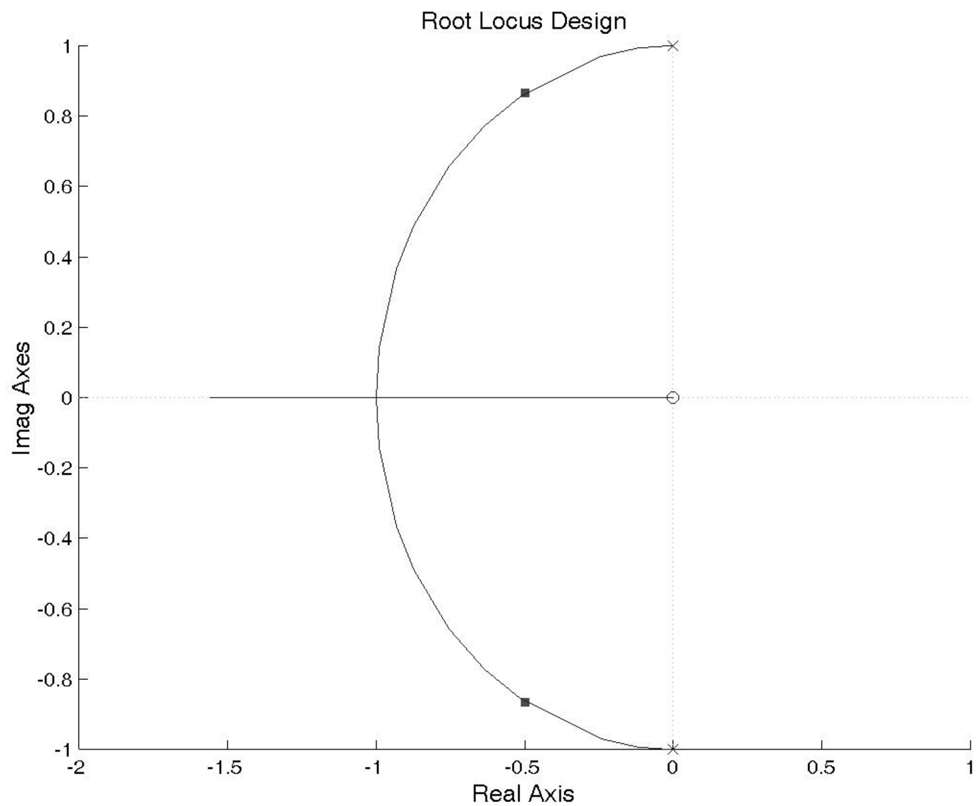
Therefore the new open-loop transfer function now is given by

$$G_1(s) = \frac{s}{s^2+1}.$$

The root locus is now drawn for the above mentioned open loop transfer function by following the same guidelines as given in example 1. The root locus can also be verifies using MATLAB using the following sequence.

```
s=tf('s');
sys = s/(s^2+1);
rltool(sys)
```

The root locus plot is given below.



Recommended Reading

"Feedback Control of Dynamic Systems" 4^{th} Edition, by Gene F. Franklin et.al – pp 270 – 307.

Recommended Assignment

"Feedback Control of Dynamic Systems" 4^{th} Edition, by Gene F. Franklin et.al – problems 5.3, 5.7a, 5.7c, 5.8a,