HANDOUT E.28 - EXAMPLE HANDOUT ON COMPENSATION USING ROOT LOCUS

Example 1:

Design a lead compensation for the system given by the transfer function

$G(s) = \dfrac{1}{s(s+1)}$, that will provide a closed-loop damping $\zeta > 0.5$ and natural frequency $\omega_n > 7$ rad/sec.

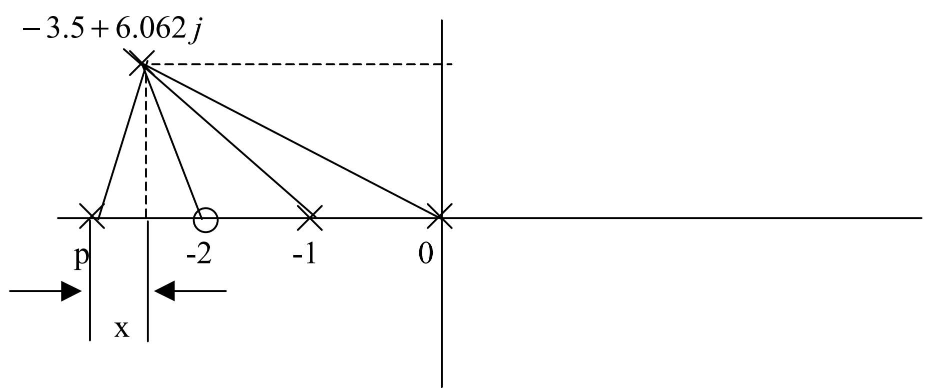
Sol: The general transfer function of a lead compensator is given as

$$D(s) = K\frac{(s+z)}{(s+p)}, \quad p > z.$$

Let us design for the limiting condition of the damping ratio and natural frequency. Therefore let us choose $\zeta = 0.5$ and $\omega_n = 7$ rad/sec.
Hence the closed loop poles of the system are given by

$$-\zeta\ \omega_n \pm j\omega_n\sqrt{1-\zeta^2},$$
$$= -3.5 \pm j6.062$$

Let us choose z = 2. The angle subtended by all the poles and zeros of the feed forward transfer function, with the closed loop pole at $-3.5+6.062j$ is



$-\theta_p - 112.41 - 120 + 103.898$. This angle must be equal to –180 degrees. Hence the angle subtended by the compensator pole with the closed loop pole is 51.488 degrees.

Therefore,

$$\tan(51.488) = \frac{6.062}{x},$$
$$\Rightarrow x = 4.824.$$

p = x + 3.5 = 4.824 + 3.5 = 8.324 ≈ 9.

To calculate the gain K, we have

$$1 + D(s)G(s) = 0,$$
$$\Rightarrow \left|D(s)G(s)\right| = 1$$
$$\Rightarrow \left|K\frac{(s+2)}{(s+9)}\frac{1}{s(s+1)}\right|_{s=-3.5+6.062j} = 1,$$
$$\Rightarrow K \approx 60.$$

Hence the lead compensator is given by

$$D(s) = 60\frac{(s+2)}{(s+9)}.$$

Plotting the root locus of the feed forward transfer function given by D(s)*G(s), the specification of the location of the location of the closed-loop pole can be verified.

Example 4:

Consider the system whose feed forward transfer function is given by

$G(s) = \frac{K}{s(s+2)}$. Design a lag compensator so that the dominant poles of the closed loop system are located at s = -1 ± j and the steady state error to a unit ramp input is less than 0.2.

Sol:

The general transfer function for lag compensation is given by

$$D(s) = \frac{(s+z)}{(s+p)}, \quad p < z$$

The forward transfer function is given as

$$G(s) * D(s) = \frac{K}{s(s+2)} \times \frac{(s+z)}{(s+p)}.$$

For the specification that the steady state error of the system must not exceed 0.2, we have

$$E(s) = \frac{s(s+2)(s+p)}{s(s+2)(s+p) + K(s+z)} R(s),$$

$$\Rightarrow e_{ss} = \lim_{s \to 0} sE(s) = \lim_{s \to 0} s \left[\frac{s(s+2)(s+p)}{s(s+2)(s+p) + K(s+z)} R(s) \right].$$

For a ramp input, we have

$$e_{ss} = \lim_{s \to 0} s \left[\frac{s(s+2)(s+p)}{s(s+2)(s+p) + K(s+z)} \right] \frac{1}{s^2} = \frac{2p}{Kz} < 0.2.$$

Let $\frac{2p}{Kz} = 0.2$

Let us choose p = 0.01, therefore we have

$$Kz = 0.1$$

We know that, the closed loop poles lie in the root locus and hence

$$1 + D(s)G(s) = 0,$$

$$\Rightarrow K = - \left. \frac{1}{D(s)G(s)} \right|_{s=-1+1j},$$

$$\Rightarrow K = - \left. \frac{s(s+2)(s+0.01)}{(s+z)} \right|_{s=-1+j}.$$

Solving for K and z, we get

K = 1.88 and since Kz = 0.1, we get z = 0.0532.

Therefore the lag compensator is given by

$$D(s) = \frac{(s+0.0532)}{(s+0.01)}.$$

Plotting the root locus of the feed forward transfer function, given by D(s)*G(s), the specification of the location of the closed loop pole can be verified.

Recommended Reading

"Feedback Control of Dynamic Systems" 4th Edition, by Gene F. Franklin et.al – pp 310 - 328.

Recommended Assignment

"Feedback Control of Dynamic Systems" 4th Edition, by Gene F. Franklin et.al – problems 5.26, 5.27.