

HANDOUT A.6 - USE OF EIGENVALUES IN SYSTEM STABILITY DETERMINATION

To determine the stability of a system using eigenvalues

Let the model of a dynamic system in the state space form be given as

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t)$$

$$y(t) = \mathbf{C}\mathbf{x}(t) + Du(t)$$

where $\mathbf{A}$, a $n \times n$ matrix, $\mathbf{B}$, a $n \times 1$ matrix $\mathbf{C}$, a $1 \times n$ matrix and D , a scalar are the quantities defining the system, ' $\mathbf{x}$ ' is a $n \times 1$ time dependent state vector, ' y ' is the time dependent output scalar and ' u ' is the time dependent input scalar. Usually it is standard practice to drop of the time dependence of ' $\mathbf{x}$ ', ' y ' and ' u ' of ' t ', while representing them in the state space equations.

The system is said to be stable if and only if all the poles of the system lie in the left half plane. In other words, if the real part of all the poles should be less than zero. The poles are nothing but the eigenvalues of the matrix $\mathbf{A}$. So if the real part of the eigenvalues of the matrix $\mathbf{A}$ is negative, then the system is stable.

Example 4: Suppose the $\mathbf{A}$ matrix for the above defined system is given by

$$\mathbf{A} = \begin{bmatrix} -9 & 1 & 0 \\ -26 & 0 & 1 \\ -24 & 0 & 0 \end{bmatrix}$$

then by calculating the eigenvalues of the matrix, one can determine the stability of the system.

Using the “*eig*” command in MATLAB, the eigenvalues are calculated as follows:

```
A = [-9 1 0;-26 0 1;-24 0 0];
[v,d] = eig(A)
```

By running the above code, the result obtained is

```
v =
    0.1270    -0.0995    0.0718
    0.6350    -0.5970    0.5026
    0.7620    -0.7960    0.8615

d =
   -4.0000         0         0
         0   -3.0000         0
         0         0   -2.0000
```

Where, the diagonal elements of the matrix '**d**' represent the eigenvalues and the column vectors of the matrix '**v**' represent the corresponding eigenvectors. From the above result it can be seen that, all the eigenvalues are real and negative. This indicates that the poles of the system are all less than zero and lie in the left half plane. Hence the system is stable.

Assignment

1) The state space form of a dynamic system is as shown below

$$\dot{\mathbf{x}} = \begin{bmatrix} 1 & 6 & -3 \\ -1 & -1 & 1 \\ -2 & -2 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = [1 \quad 0 \quad 0] \mathbf{x}$$

a) Determine the eigenvalues of this system.

b) Is the system stable? Give reasons.