

HANDOUT A.2 - LAPLACE TRANSFORMS

NOTE: All the transformations have to be done using the analytical method outlined. MATLAB has to be used only to verify the result obtained.

Introduction

The Laplace transform is the mathematical tool that can be used for transforming differential equations into an easier-to-manipulate algebraic form. The advantages of this modern transform method for the analysis of linear-time-invariant (LTI) systems are the following:

1. It includes the boundary or initial conditions.
2. The mathematics involved in the solution is simple algebra.
3. The work is systematized.
4. The use of table of transforms reduces the required labor.
5. Discontinuous inputs can be treated.
6. The transient and the steady-state components of the solution are obtained simultaneously.

The disadvantage of transform methods is that if they are used mechanically without the knowledge of the actual theory involved, they sometimes yield erroneous results.

Definition of the Laplace transform

The direct Laplace transformation of a function of time $f(t)$ is given by

$$L[f(t)] = \int_0^{\infty} f(t) e^{-st} dt = F(s) \quad (1)$$

where $L[f(t)]$ is a shorthand notation for the Laplace integral. Evaluation of the integral results in a function $F(s)$ that has 's' as the parameter. This parameter 's' is a complex quantity of the form $a + bi$.

Derivations of Laplace transforms for simple functions

A number of examples are presented to show the derivation of the Laplace transform for several time functions. A list of common transform pairs is given at the end of the handout.

Example 1*Step Function*

The step function of size 'a' is defined as follows:

$$u(t) = \begin{cases} a & 0 \leq t < \infty \\ 0 & -\infty < t < 0 \end{cases}$$

The Laplace transform of the above defined step function is obtained by substituting the function in equation (1).

$$L[u(t)] = \int_0^{\infty} u(t)e^{-st} dt = U(s)$$

since $u(t)$ has the value of 'a' over the limits of integration,

$$U(s) = \int_0^{\infty} ae^{-st} dt = \left. \frac{-ae^{-st}}{s} \right|_0^{\infty} = \frac{a}{s}$$

Decaying exponential function $e^{-\alpha t}$

The Laplace transform of the above-mentioned exponential function is

$$L[e^{-\alpha t}] = \int_0^{\infty} e^{-\alpha t} e^{-st} dt = \int_0^{\infty} e^{-(s+\alpha)t} dt = \left. \frac{-e^{-(s+\alpha)t}}{s+\alpha} \right|_0^{\infty} = \frac{1}{s+\alpha}$$

Using MATLAB to calculate the Laplace transform

To find the Laplace transforms of functions using MATLAB, use the '*laplace*' command. To know more about the command, type the following command in the MATLAB command window.

```
help laplace
```

```
--- help for sym/laplace.m ---
```

```
LAPLACE Laplace transform.
```

```
L = LAPLACE(F) is the Laplace transform of the scalar sym F with
default independent variable t. The default return is a function
of s. If F = F(s), then LAPLACE returns a function of t: L = L(t).
By definition L(s) = int(F(t)*exp(-s*t),0,inf), where integration
occurs with respect to t.
```

`L = LAPLACE(F,t)` makes `L` a function of `t` instead of the default `s`:
`LAPLACE(F,t) <=> L(t) = int(F(x)*exp(-t*x),0,inf)`.

`L = LAPLACE(F,w,z)` makes `L` a function of `z` instead of the default `s` (integration with respect to `w`).
`LAPLACE(F,w,z) <=> L(z) = int(F(w)*exp(-z*w),0,inf)`.

Examples:

```
syms a s t w x
laplace(t^5)           returns 120/s^6
laplace(exp(a*s))      returns 1/(t-a)
laplace(sin(w*x),t)    returns w/(t^2+w^2)
laplace(cos(x*w),w,t)  returns t/(t^2+x^2)
laplace(x^sym(3/2),t)  returns 3/4*pi^(1/2)/t^(5/2)
laplace(diff(sym('F(t)')) returns laplace(F(t),t,s)*s-F(0)
```

See also `ILAPLACE`, `FOURIER`, `ZTRANS`.

Note that there are also a few examples in the help file. The '*laplace*' command is further explained with the help of the following examples.

Example 2

1. Determine the Laplace transform of the following step function using MATLAB.

$$u(t) = \begin{cases} 2 & 0 \leq t < \infty \\ 0 & -\infty < t < 0 \end{cases}$$

The MATLAB code is as follows.

```
syms t;
f = 2*(t^0);
ans = laplace(f)
```

Note that the '*syms*' command in the first statement of the code implies that the variable '*t*' is to be considered as a symbol. The '*laplace*' command works if and only if the argument is a function of time. Since the function defined in the example is a constant, it is converted to a function of time by multiplying the constant with '*t*' to the power zero. The result of the above code is

ans =

2/s

The answer can be verified by following the procedure outlined in example 1.

2. Determine the Laplace transform of the following ramp function using MATLAB.

$$f(t) = \begin{cases} bt & 0 \leq t < \infty \\ 0 & -\infty < t < 0 \end{cases}$$

Analytical method

$$\begin{aligned} F(s) &= \int_0^{\infty} f(t)e^{-st} dt = \int_0^{\infty} bte^{-st} dt \\ &\Rightarrow b \int_0^{\infty} te^{-st} dt = bt \frac{e^{-st}}{(-s)} \Big|_0^{\infty} - b \frac{1}{(-s)} \int_0^{\infty} (1)e^{-st} dt \\ &= 0 + \frac{b}{s} \int_0^{\infty} e^{-st} dt = \frac{b}{s} \frac{e^{-st}}{(-s)} \Big|_0^{\infty} = 0 - \frac{b}{s(-s)} = \frac{b}{s^2} \end{aligned}$$

The integration by parts technique is used to obtain the above integral.

The MATLAB code to verify the result obtained is

```
syms b,t;
f = b*t;
ans = laplace(f)
```

Again note that, the variable 'b' also has to be defined as a symbolic variable. The result is

```
ans =
b/s^2
```

Properties of Laplace transforms

1. *Linearity.* If 'a' is a constant or is independent of 's' and 't', and if $f(t)$ is transformable, then

$$L[af(t)] = aL[f(t)] = aF(s)$$

2. *Superposition.* If $f_1(t)$ and $f_2(t)$ are both Laplace-transformable, the principle of superposition applies:

$$L[f_1(t) \pm f_2(t)] = L[f_1(t)] \pm L[f_2(t)] = F_1(s) \pm F_2(s)$$

3. *Translation in time.* If the Laplace transform of $f(t)$ is $F(s)$ and 'a' is a positive real number, then the Laplace transform of the translated function $f(t-a)$ is

$$L[f(t-a)] = e^{-as} F(s)$$

4. *Complex Differentiation.* If the Laplace transform of $f(t)$ is $F(s)$, then

$$L[tf(t)] = -\frac{d}{ds} F(s)$$

Multiplication by time in the time domain entails differentiation with respect to 's' in the s-domain.

5. *Translation in the s domain.* If the Laplace transform of $f(t)$ is $F(s)$ and 'a' is either real or complex, then

$$L[e^{at} f(t)] = F(s-a)$$

6. *Real Differentiation.* If the Laplace transform of $f(t)$ is $F(s)$ and if the first derivative of $f(t)$ with respect to time $Df(t)$ is transformable, then

$$L[Df(t)] = sF(s) - f(0^+)$$

The term $f(0^+)$ is the value of the right-hand limit of the function $f(t)$ as the origin $t = 0$ is approached from the right side (thus through positive value of time). For simplicity the plus sign following the zero is omitted, although its presence is implied.

The transform of the second derivative $D^2f(t)$ is

$$L[D^2 f(t)] = s^2 F(s) - sf(0) - Df(0)$$

where $Df(0)$ is the value of the limit of the derivative of $f(t)$ as the origin $t = 0$, is approached from the right side.

7. *Final value Theorem.* If $f(t)$ and $Df(t)$ are Laplace-transformable, if the Laplace transform of $f(t)$ is $F(s)$ and if $\lim_{t \rightarrow \infty} f(t)$ exists, then

$$\lim_{s \rightarrow 0} sF(s) = \lim_{t \rightarrow \infty} f(t)$$

Example

Find the steady state value of the system corresponding to

$$Y(s) = \frac{3}{s(s+2)}$$

Solution

From the final value theorem, the steady state value of the function is given by

$$\lim_{t \rightarrow \infty} y(t) = \lim_{s \rightarrow 0} sY(s) = \lim_{s \rightarrow 0} s \left(\frac{3}{s(s+2)} \right) = \lim_{s \rightarrow 0} \frac{3}{s+2} = \frac{3}{2}$$

Thus, after the transients have decayed to zero, $y(t)$ will settle to a constant value of 1.5.

8. *Initial value Theorem.* If the function $f(t)$ and its first derivative are Laplace transformable, if the Laplace transform of $f(t)$ is $F(s)$, and if $\lim_{s \rightarrow \infty} sF(s)$ exists, then

$$\lim_{s \rightarrow \infty} sF(s) = \lim_{t \rightarrow 0} f(t)$$

Applications of the Laplace transform to differential equation

Let us consider a simple mass-spring-damper system, whose equation of motion is given by

$$m \ddot{x} + c \dot{x} + kx = f(t) \quad (2)$$

The unknown quantity for which the equation is to be solved is the displacement of the mass ' $x(t)$ '. The above equation can be easily solved using the Laplace transform. $f(t)$ is the forcing function.

Now

$$L[f(t)] = F(s);$$

$$L[m \ddot{x}] = m[s^2 X(s) - sx(0) - \dot{x}(0)];$$

$$L[c \dot{x}] = c[sX(s) - x(0)];$$

$$L[kx] = kX(s)$$

The above results are obtained based on property (6).

Substituting the above results in equation (2) we have

$$m[s^2 X(s) - sx(0) - \dot{x}(0)] + c[sX(s) - x(0)] + kX(s) = F(s);$$

$$\Rightarrow (ms^2 + cs + k)X(s) - (ms + c)x(0) - m\dot{x}(0) = F(s);$$

From the above equation,
we can see that $x(0)$ and

$\dot{x}(0)$ are the initial conditions for the displacement and the velocity. If both the initial conditions are equal to zero, then the above equation reduces to

$$(ms^2 + cs + k)X(s) = F(s);$$

$$\Rightarrow X(s) = \frac{F(s)}{(ms^2 + cs + k)}$$

Based on the forcing function, the Laplace transform of $f(t)$ can be easily found, as a result of which the value of $X(s)$ can be found. In order to get the value of the displacement in the time-domain, we need to determine the inverse Laplace transform of $X(s)$ to get $x(t)$. To get the inverse Laplace transform, the above relation is expanded using partial fractions and then the inverse Laplace transform is obtained by looking in the table given in page 733 in “Feedback control of dynamic systems. Third Edition by Franklin et.al.”

To get the inverse Laplace transform using MATLAB, use the ‘*ilaplace*’ function in MATLAB. By typing the following command in the command window yields the result:

```
help ilaplace
```

```
--- help for sym/ilaplace.m ---
```

```
ILAPLACE Inverse Laplace transform.
```

```
F = ILAPLACE(L) is the inverse Laplace transform of the scalar sym
L
with default independent variable s. The default return is a
function of t. If L = L(t), then ILAPLACE returns a function of x:
F = F(x).
```

```
By definition, F(t) = int(L(s)*exp(s*t),s,c-i*inf,c+i*inf)
where c is a real number selected so that all singularities
of L(s) are to the left of the line s = c, i = sqrt(-1), and
the integration is taken with respect to s.
```

```
F = ILAPLACE(L,y) makes F a function of y instead of the default t:
ILAPLACE(L,y) <=> F(y) = int(L(y)*exp(s*y),s,c-i*inf,c+i*inf).
Here y is a scalar sym.
```

```
F = ILAPLACE(L,y,x) makes F a function of x instead of the default
t:
ILAPLACE(L,y,x) <=> F(y) = int(L(y)*exp(x*y),y,c-i*inf,c+i*inf),
integration is taken with respect to y.
```

```
Examples:
```

```
syms s t w x y
ilaplace(1/(s-1))           returns    exp(t)
```

<code>ilaplace(1/(t^2+1))</code>	<code>returns</code>	<code>sin(x)</code>
<code>ilaplace(t^(-sym(5/2)),x)</code>	<code>returns</code>	<code>4/3/pi^(1/2)*x^(3/2)</code>
<code>ilaplace(y/(y^2 + w^2),y,x)</code>	<code>returns</code>	<code>cos(w*x)</code>
<code>ilaplace(sym('laplace(F(x),x,s)'),s,x)</code>	<code>returns</code>	<code>F(x)</code>

Example 3

Determine the inverse transform of the function

$$X(s) = \frac{2}{s+2}$$

Analytical method

By looking at the table of Laplace transforms on page 733 of the “Feedback control of dynamic systems. Third edition by Franklin et.al” it can be seen that, the inverse Laplace

transform of $\frac{1}{(s+a)}$ is e^{-at} . So the inverse Laplace transform of the above problem is

$$2e^{-2t}.$$

The MATLAB code to verify the above result is

```
syms s;
f = 2/(s+2);
ans = ilaplace(f)
```

Note that, the variable ‘s’ is defined as a symbolic variable. The result of the above code is

```
ans =
2*exp(-2*t)
```

In other words the result is $2e^{-2t}$.

Example 4

Solve the differential equation given by

$$\ddot{y}(t) + y(t) = 0 \tag{3}$$

The initial conditions are

$$y(0) = \alpha$$

$$\dot{y}(0) = 0;$$

Solution

First let us define the Laplace transform of each of the individual terms in the equation.

$$L[\ddot{y}(t)] = s^2 Y(s) - sy(0) - \dot{y}(0)$$

$$L[y(t)] = Y(s);$$

Substituting the initial conditions in the above equations, we have

$$L[\ddot{y}(t)] = s^2 Y(s) - s\alpha$$

Substituting the Laplace transforms in equation (3), we have

$$s^2 Y(s) - s\alpha + Y(s) = 0$$

$$\Rightarrow Y(s) = \frac{s\alpha}{(s^2 + 1)}$$

After looking up in the transform tables, the inverse laplace transform of Y(s) can be obtained as

$$y(t) = \alpha \cos t .$$

It is always better to remember some of the inverse Laplace transform formulae.

Partial fraction expansion theorems

The partial fractions technique is used when one needs to find the inverse Laplace transform. For example consider the Laplace transform of a function to be

$$F(s) = \frac{P(s)}{Q(s)} = \frac{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0}{s^m + b_{n-1} s^{n-1} + \cdots + b_1 s + b_0}$$

The a's and b's are real constants and the coefficient of the highest power of 's' in the denominator has been made equal to unity. The first step is to factor Q(s) into first-order and quadratic factors with real coefficients:

$$F(s) = \frac{P(s)}{Q(s)} = \frac{P(s)}{(s - s_1)(s - s_2) \cdots (s - s_k) \cdots (s - s_n)}$$

The values $s_1, s_2, \dots, s_n$ that make the denominator equal to zero are called the *roots* of the denominator. These values of s, may be either real or complex. To calculate the roots of Q(s), Q(s) is equated to zero, i.e.,

$$Q(s) = 0;$$

$$\Rightarrow s^m + b_{n-1}s^{n-1} + \dots + b_1s + b_0 = 0$$

The transform $F(s)$ can be expressed as a series of fractions:

$$F(s) = \frac{P(s)}{Q(s)} = \frac{A_1}{s-s_1} + \frac{A_2}{s-s_2} + \dots + \frac{A_n}{s-s_n}$$

The procedure is to evaluate the constants $A_1, A_2, \dots, A_n$ corresponding to the poles $s_1, s_2, \dots, s_n$. The coefficients $A_1, A_2, \dots, A_n$ are termed as the residues of $F(s)$ at the corresponding poles.

There are four cases of problems, depending on the denominator $Q(s)$:

- *Case 1:* $F(s)$ has first order real poles.
- *Case 2:* $F(s)$ has repeated first order real poles.
- *Case 3:* $F(s)$ has a pair of complex conjugate poles (a quadratic factor in the denominator).
- *Case 4:* $F(s)$ has repeated pairs of complex conjugate poles (a repeated quadratic factor in the denominator.)

Case 1: First order real poles

Consider the Laplace transform

$$F(s) = \frac{P(s)}{Q(s)} = \frac{P(s)}{s(s-s_1)(s-s_2)} = \frac{A_0}{s} + \frac{A_1}{s-s_1} + \frac{A_2}{s-s_2} \quad (4)$$

where s_1 and s_2 may be positive or negative or zero.

To evaluate a typical coefficient A_k , multiply both sides of equation (4) by $(s-s_1)$, the result is

$$\begin{aligned} (s-s_1)F(s) &= (s-s_1) \frac{P(s)}{Q(s)} = (s-s_1) \frac{P(s)}{s(s-s_1)(s-s_2)} \\ \Rightarrow \frac{P(s)}{s(s-s_2)} &= A_0 \frac{s-s_1}{s} + A_1 + A_2 \frac{s-s_1}{s-s_2} \end{aligned}$$

The multiplying factor $(s-s_1)$ on the left side of the equation and the same factor of $Q(s)$ should be cancelled. By letting $s = s_1$, each term on the right hand side of the equation is zero except A_1 . Thus, a general rule for evaluating the constants for single order poles is

$$A_k = \left[(s - s_k) \frac{P(s)}{Q(s)} \right]_{s=s_k}$$

Example 5

For example

$$F(s) = \frac{(s+2)}{s(s+1)(s+3)} = \frac{A_0}{s} + \frac{A_1}{s+1} + \frac{A_2}{s+3}$$

$$A_0 = [sF(s)]_{s=0} = \left[\frac{s+2}{(s+1)(s+3)} \right]_{s=0} = \frac{2}{3}$$

$$A_1 = [(s+1)F(s)]_{s=-1} = \left[\frac{s+2}{s(s+3)} \right]_{s=-1} = -\frac{1}{2}$$

$$A_2 = [(s+3)F(s)]_{s=-3} = \left[\frac{s+2}{s(s+1)} \right]_{s=-3} = -\frac{1}{6}$$

Hence the partial transform expansion is

$$F(s) = \frac{2}{3} \frac{1}{s} + \frac{-1}{2} \frac{1}{s+1} + \frac{-1}{6} \frac{1}{s+3}$$

$$\Rightarrow F(s) = \frac{0.6667}{s} - \frac{0.5}{s+1} - \frac{0.1667}{s+3}$$

Case 2: Multiple order poles

For the general transform with repeated real roots

$$F(s) = \frac{P(s)}{Q(s)} = \frac{P(s)}{(s - s_q)^r (s - s_1) \cdots}$$

$$\Rightarrow F(s) = \frac{A_{qr}}{(s - s_q)^r} + \frac{A_{q(r-1)}}{(s - s_q)^{r-1}} + \cdots + \frac{A_{q1}}{(s - s_q)} + \frac{A_1}{(s - s_1)} + \cdots$$

The coefficient A_{qr} can be obtained by following the procedure outlined in the previous case, i.e.,

$$A_{qr} = \left[(s - s_q)^r \frac{P(s)}{Q(s)} \right]_{s=s_q}$$

To determine $A_{q(r-1)}$, differentiate A_{qr} with respect to 's' and then substitute the value of 's' = s_q , i.e.,

$$A_{q(r-1)} = \left\{ \frac{d}{ds} \left[(s - s_q)^r \frac{P(s)}{Q(s)} \right] \right\}_{s=s_q}$$

Repeating the differentiation gives the coefficient $A_{q(r-2)}$ as

$$A_{q(r-2)} = \left\{ \frac{1}{2} \frac{d^2}{ds^2} \left[(s - s_q)^r \frac{P(s)}{Q(s)} \right] \right\}_{s=s_q}$$

In general

$$A_{q(r-k)} = \left\{ \frac{1}{k!} \frac{d^k}{ds^k} \left[(s - s_q)^r \frac{P(s)}{Q(s)} \right] \right\}_{s=s_q}$$

Example 6

$$F(s) = \frac{1}{(s+2)^3 (s+3)} = \frac{A_{13}}{(s+2)^3} + \frac{A_{12}}{(s+2)^2} + \frac{A_{11}}{(s+2)} + \frac{A_2}{(s+3)}$$

The constants are

$$A_{13} = \left[(s+2)^3 F(s) \right]_{s=-2} = \left[(s+2)^3 \frac{1}{(s+2)^3 (s+3)} \right]_{s=-2} = 1$$

$$A_{12} = \left\{ \frac{d}{ds} \left[(s+2)^3 F(s) \right] \right\}_{s=-2} = \left\{ \frac{d}{ds} \left[\frac{1}{(s+3)} \right] \right\}_{s=-2} = \left[\frac{-1}{(s+3)^2} \right]_{s=-2} = -1$$

$$A_{11} = \left\{ \frac{1}{2} \frac{d^2}{ds^2} \left[(s+2)^3 F(s) \right] \right\}_{s=-2} = \left\{ \frac{1}{2} \frac{d^2}{ds^2} \left[\frac{1}{(s+3)} \right] \right\}_{s=-2} = \left\{ \frac{1}{2} \frac{d}{ds} \left[\frac{-1}{(s+3)^2} \right] \right\}_{s=-2} = \left[\frac{1}{2} \frac{2}{(s+3)^3} \right]_{s=-2} = 1$$

$$A_2 = \left[(s+3) F(s) \right]_{s=-3} = \left[\frac{1}{(s+2)^3} \right]_{s=-3} = -1$$

Therefore

$$F(s) = \frac{1}{(s+2)^3} - \frac{1}{(s+2)^2} + \frac{1}{(s+2)} - \frac{1}{(s+3)}$$

Case 3: Complex conjugate poles

The procedure is the same as that of case 1.

Case 4: Multiple order complex poles

The procedure is the same as that of case 2.

Using MATLAB to calculate the partial fractions

The 'residue' function in MATLAB is used to obtain the coefficients and the poles of the transform. The online help gives

help residue

```
RESIDUE Partial-fraction expansion (residues).
[R,P,K] = RESIDUE(B,A) finds the residues, poles and direct term of
a partial fraction expansion of the ratio of two polynomials
B(s)/A(s).
If there are no multiple roots,
      B(s)      R(1)      R(2)      ...      R(n)
      ---- = ---- + ---- + ... + ---- + K(s)
      A(s)      s - P(1)  s - P(2)      s - P(n)
Vectors B and A specify the coefficients of the numerator and
denominator polynomials in descending powers of s. The residues
are returned in the column vector R, the pole locations in column
vector P, and the direct terms in row vector K. The number of
```

poles is $n = \text{length}(A) - 1 = \text{length}(R) = \text{length}(P)$. The direct term coefficient vector is empty if $\text{length}(B) < \text{length}(A)$, otherwise $\text{length}(K) = \text{length}(B) - \text{length}(A) + 1$.

If $P(j) = \dots = P(j+m-1)$ is a pole of multiplicity m , then the expansion includes terms of the form

$$\frac{R(j)}{s - P(j)} + \frac{R(j+1)}{(s - P(j))^2} + \dots + \frac{R(j+m-1)}{(s - P(j))^m}$$

$[B,A] = \text{RESIDUE}(R,P,K)$, with 3 input arguments and 2 output arguments,

converts the partial fraction expansion back to the polynomials with coefficients in B and A.

Warning: Numerically, the partial fraction expansion of a ratio of polynomials represents an ill-posed problem. If the denominator polynomial, $A(s)$, is near a polynomial with multiple roots, then small changes in the data, including roundoff errors, can make arbitrarily large changes in the resulting poles and residues. Problem formulations making use of state-space or zero-pole representations are preferable.

The use of the command is explained with the help of an example.

Example 7

For the transform given calculate the poles and the coefficients of the partial fraction expansion.

$$F(s) = \frac{(s+2)}{(s^3 + 4s^2 + 3s)}$$

Note that, in the residue command the coefficients of the numerator and the denominator should be given in the form of a matrix.

Analytical method

$$F(s) = \frac{(s+2)}{s(s^2 + 4s + 3)} = \frac{(s+2)}{s(s+3)(s+1)} = \frac{A_1}{s} + \frac{A_2}{(s+3)} + \frac{A_3}{(s+1)}$$

$$A_1 = [sF(s)]_{s=0} = \left[\frac{(s+2)}{(s+3)(s+1)} \right]_{s=0} = \frac{2}{3} = 0.6667$$

$$A_2 = [(s+3)F(s)]_{s=-3} = \left[\frac{(s+2)}{s(s+1)} \right]_{s=-3} = \frac{(-1)}{(-3)(-2)} = -\frac{1}{6} = -0.1667$$

$$A_3 = [(s+1)F(s)]_{s=-1} = \left[\frac{(s+2)}{s(s+3)} \right]_{s=-1} = \frac{1}{(-1)(2)} = -\frac{1}{2} = -0.5$$

Therefore

$$F(s) = \frac{0.6667}{s} - \frac{0.1667}{(s+3)} - \frac{0.5}{(s+1)}$$

The MATLAB code verify the above result obtained is

```
numerator = [1 2];
denominator = [1 4 3 0];
[r,p,k] = residue(numerator, denominator)
```

The result of the above code is

r =

```
-0.1667
-0.5000
 0.6667
```

p =

```
-3
-1
 0
```

k =

```
[]
```

In the above result, 'r' denotes the coefficients ' A_1 ', ' A_2 ' and ' A_3 ' and 'p' indicates the poles of the system

Assignment

1) Determine the Laplace transform of the following functions.

a) $\sin(at)$

b) t^3

c) $4t^5$

d) te^{-at}

Verify the results using MATLAB.

2) Determine the Inverse Laplace transform of the following functions

a) $\frac{3}{s^2}$

b) $\frac{4}{s+5}$

c) $\frac{1}{(s+a)^2}$

Verify the result using MATLAB.

3) Solve the following differential equation

$$\ddot{y}(t) + 4y(t) = 0$$

Given the initial conditions

$$y(0) = 0$$

$$\dot{y}(0) = \beta$$

Recommended Reading

“Feedback Control of Dynamic systems” Fourth Edition by Gene F. Franklin et.al.
pp 96 – 115.

Recommended Assignment

“Feedback Control of Dynamic systems” Fourth Edition by Gene F. Franklin et.al.

pp – 182 ----3.2a, 3.2c, 3.3a, 3.4b, 3.4c, 3.5a, 3.5b, 3.7b, 3.7e.

pp – 183 ----3.9