

Texas A & M University
Department of Mechanical Engineering
MEEN 364 Dynamic Systems and Controls
Dr. Alexander G. Parlos

Lecture 18: Basic Properties of Feedback
A Case Study of Speed Control

The objective of this lecture is to present some of the most fundamental elements of control loops, including open-loop and closed-loop (or feedback) control. There are two basis structures for control of dynamic systems: (a) open-loop control and (b) feedback (or closed-loop) control. These two control structures are shown in Figures 1 and 2.

Case Study of Motor Speed Control

Let us write the equations of motion for a DC motor coupled to an inertial load. The electrical dynamics can be expressed as

$$K_e \dot{\theta}_m(t) + L_a \frac{di_a(t)}{dt} + R_a i_a(t) = v_a(t), \quad (1)$$

whereas the mechanical dynamics can be expressed as

$$J_m \ddot{\theta}_m(t) + b \dot{\theta}_m(t) = K_t i_a(t) + T_\ell. \quad (2)$$

Let us define the output to the motor speed $y(t) = \dot{\theta}_m(t)$ and let's name the motor load as the disturbance $w(t) = T_\ell$. Taking the Laplace transforms of equations (1) and (2) and eliminating the current $I_a(s)$ we obtain an equation of the form¹

$$\begin{aligned} \left(\frac{J_m L_a}{b R_a + K_t K_e} s^2 + \frac{J_m R_a + b L_a}{b R_a + K_t K_e} s + 1 \right) Y(s) &= \\ &= \frac{K_t}{b R_a + K_t K_e} V_a(s) + \frac{1}{b R_a + K_t K_e} W(s) \end{aligned} \quad (3)$$

¹NOTE: Please, do the algebra to convince yourselves of this result.

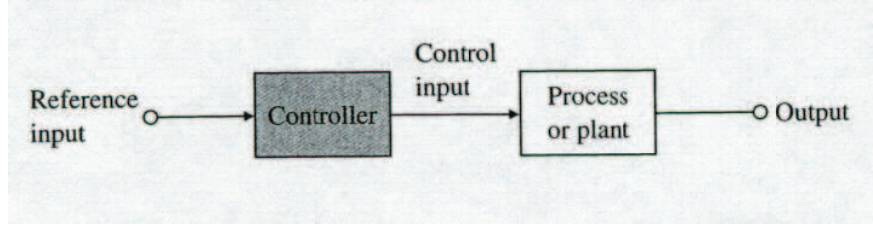


Figure 1: Open-loop Control System.

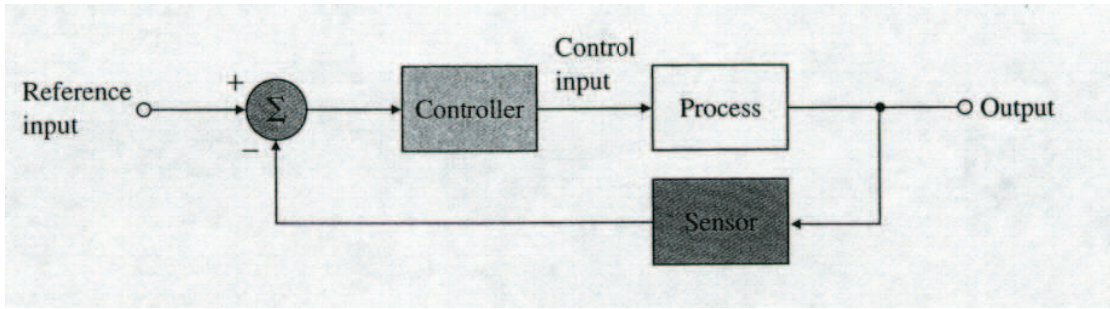


Figure 2: Closed-loop or Feedback Control System.

Equation (3) can be simplified to the form

$$(\tau_1 s + 1)(\tau_2 s + 1)Y(s) = AV_a(s) + BW(s), \quad (4)$$

where the time constants τ_1 and τ_2 , as well as the constants A and B are expressed in terms of the DC motor variables. Equation (4) can be written in a familiar transfer function form, as follows:

$$Y(s) = \frac{A}{(\tau_1 s + 1)(\tau_2 s + 1)}V_a(s) + \frac{B}{(\tau_1 s + 1)(\tau_2 s + 1)}W(s). \quad (5)$$

At steady-state, when both $w(t)$ and $v_a(t)$ are constant, we have the steady-state motor response as

$$y_{ss} = Av_a + Bw. \quad (6)$$

Figures 3 and 4 show the DC motor in open-loop and closed-loop speed control configuration.

We now compare the various properties of the two control systems we considered, i.e. open and close-loop control systems.

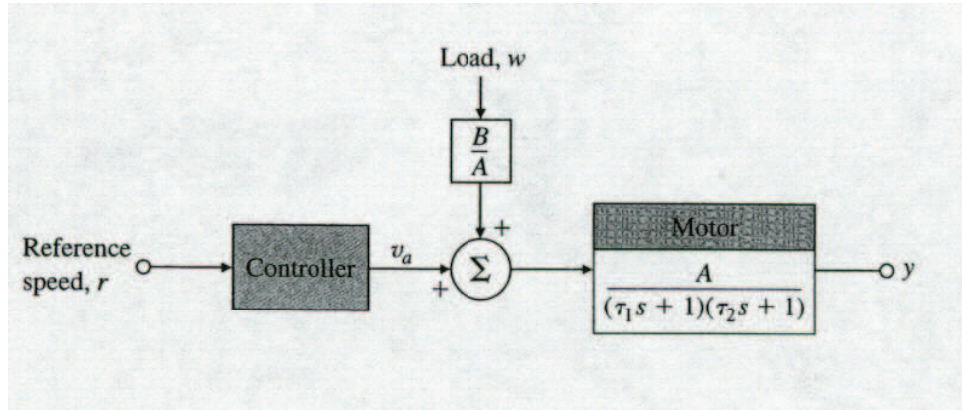


Figure 3: Open-loop DC Motor Speed Control System.

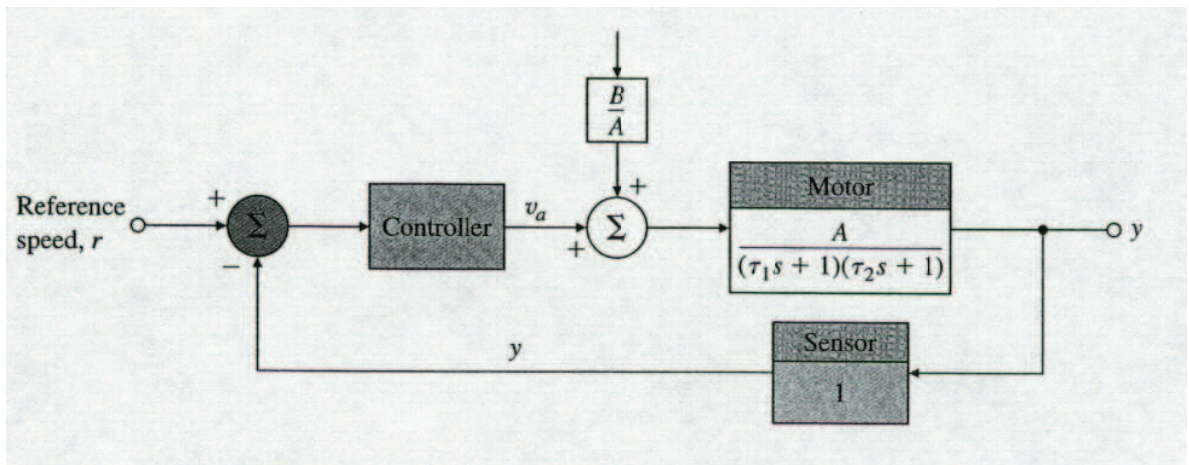


Figure 4: Closed-loop (Feedback) DC Motor Speed Control System.

Disturbance Rejection

One of the important characteristics of a speed control system is how well it maintains (regulates) speed at steady-state, in the presence of disturbances or torques. For the open-loop controller the control input (the amplifier voltage) is

$$v_a = Kr, \quad (7)$$

where the controller gain is chosen such that the motor speed is equal to the desired speed r when the torque w is zero. That is

$$K = \frac{1}{A}. \quad (8)$$

The DC motor speed with open-loop control and zero load torque will be given by

$$y_{ss} = Av_a = A\frac{1}{A}r = r. \quad (9)$$

For the case of the feedback controller the amplifier voltage will be

$$v_a = K(r - y). \quad (10)$$

In view of this control law, the closed-loop transfer function in this case will be

$$Y(s) = \frac{AK}{(\tau_1 s + 1)(\tau_2 s + 1) + AK}R(s) + \frac{B}{(\tau_1 s + 1)(\tau_2 s + 1) + AK}W(s). \quad (11)$$

At steady-state ($s \rightarrow 0$) with zero load torque we have

$$y_{ss} = \frac{AK}{1 + AK}r, \quad (12)$$

where if the gain K is selected such that $AK \gg 1$, then $y_{ss} \cong r$. So, for both control systems we obtain the desired speed if there is no load torque.

What happens when the load torque is not zero?

For the open-loop system we have that the steady-state speed is

$$y_{ss} = AKr + Bw, \quad (13)$$

or with the controller $K = \frac{1}{A}$ we have

$$y_{ss} = r + Bw, \quad (14)$$

and if we define the speed variation caused by the load torque as $\delta y = y_{ss} - r$ then

$$\delta y = Bw. \quad (15)$$

So, the speed error is proportional to the load torque, where the constant of proportionality is B and fix for a given problem!

For the closed-loop system we have the steady-state speed given by

$$y_{ss} = \frac{AK}{1 + AK}r + \frac{B}{1 + AK}w. \quad (16)$$

If the controller is designed such that $AK \gg 1$ and $AK \gg B$, then there will be no significant error in the motor speed, despite the presence of any amount of load torque.

Advantage of Feedback: Reduce Impact of Disturbances Output errors can be made less sensitive to disturbances with feedback than without feedback, by an amount of $1 + AK$.

Sensitivity to Gain Changes

Another comparison that can be made between the two controllers is that of changing the controller gain value.

Assume that temperature effects have resulted in the motor gain to shift from A to $A + \delta A$. In the open-loop case the controller gain K will still be $\frac{1}{A}$, and the new overall system gain would be

$$T_{ol} + \delta T_{ol} = K(A + \delta A) = \frac{1}{A}(A + \delta A) = 1 + \frac{\delta A}{A}, \quad (17)$$

where T_{ol} is the open-loop torque. So, the gain error is $\frac{\delta A}{A}$. In terms of percent changes, defined as $\frac{\delta T}{T}$, we have

$$\frac{\delta T_{ol}}{T_{ol}} = \frac{\delta A}{A}, \quad (18)$$

which means that a 10% error in A would translate in a 10% in T_{ol} . The ratio of $\delta T/T$ to $\delta A/A$ is called the sensitivity of the gain from r to y_{ss} , with respect to A . For the open-loop case this ratio is 1.

Applying the same change in A to the feedback case yields,

$$T_{cl} + \delta T_{cl} = \frac{(A + \delta A)K}{1 + (A + \delta A)K}. \quad (19)$$

One can compute the sensitivity using equation (19) and differential calculus. However, an easier approach is to first define the sensitivity as

$$S_A^{T_{cl}} \equiv \frac{A}{T_{cl}} \frac{dT_{cl}}{dA}, \quad (20)$$

and then apply it to the close-loop transfer function

$$T_{cl} = \frac{AK}{1 + AK}. \quad (21)$$

The result is

$$S_A^{T_{cl}} = \frac{1}{1 + AK}, \quad (22)$$

which reveals another major advantage of feedback control.

Advantage of Feedback: Reduce Impact of Uncertainties When using feedback control, output errors can be made less sensitive to variations in the plant gain A by a factor of $1 + AK$, as compared to open-loop control.

Dynamic Tracking

So far we have looked at steady-state system responses under constant disturbances and references. However, most control systems must track time-varying inputs. The differences between open-loop and closed-loop control with respect to dynamic tracking can best be seen through an example.

Consider a servomechanism with $\tau_1 = \frac{1}{60}$, $\tau_2 = \frac{1}{600}$, $A = 10$ and $B = 50$ and with the open-loop controller gain set at 0.1. Determine the output for a step change in the load torque of $-0.1 \text{ N} \cdot \text{m}$.

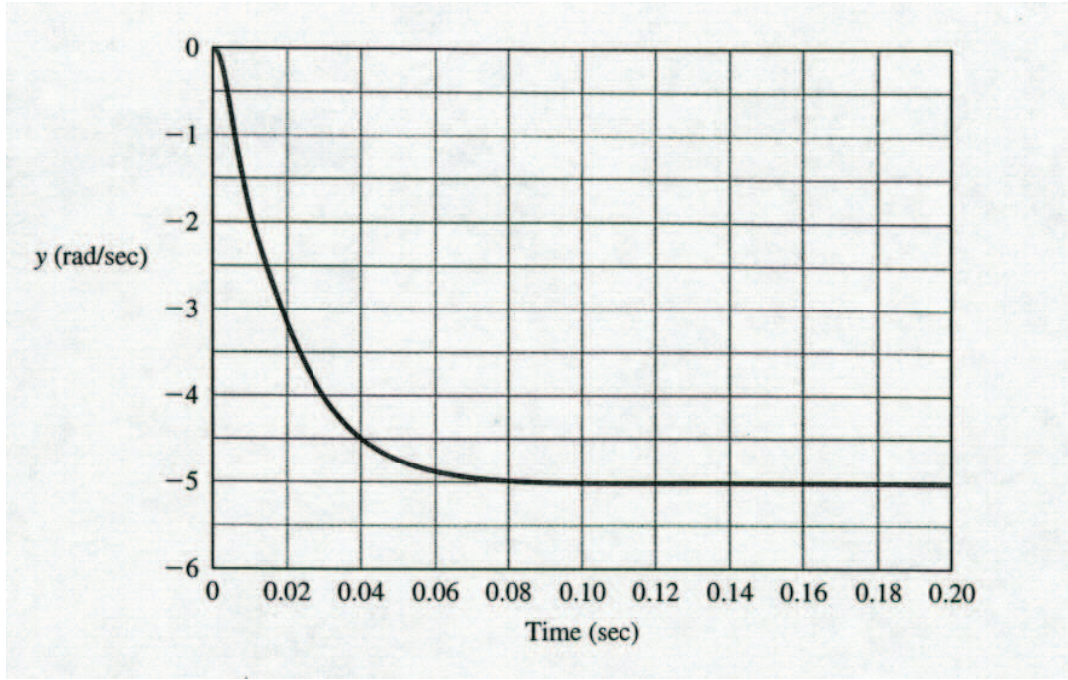


Figure 5: Open-loop transient response to a disturbance input.

For the open-loop case, assuming that $r = 0$ the output is expressed as

$$Y(s) = \frac{50}{(\frac{1}{60}s + 1)(\frac{1}{600}s + 1)}W(s). \quad (23)$$

The disturbance is given by

$$W(s) = -\frac{0.1}{s}. \quad (24)$$

So,

$$Y(s) = \frac{-5}{(\frac{1}{60}s + 1)(\frac{1}{600}s + 1)s}. \quad (25)$$

The system response is shown in Figure 5.

Now, let's assume that we wish to use feedback control in order to improve the system's ability to reject steady-state disturbances by a factor of 100 with respect to the open-loop case. The output is given by

$$Y(s) = \frac{50}{\tau_1\tau_2s^2 + (\tau_1 + \tau_2)s + (1 + AK)}W(s). \quad (26)$$

Since we want the error reduction to be 100 we must have $1 + AK = 100$. So, $K = 9.9$. Considering that $W(s) = -\frac{0.1}{s}$, from the final value theorem we

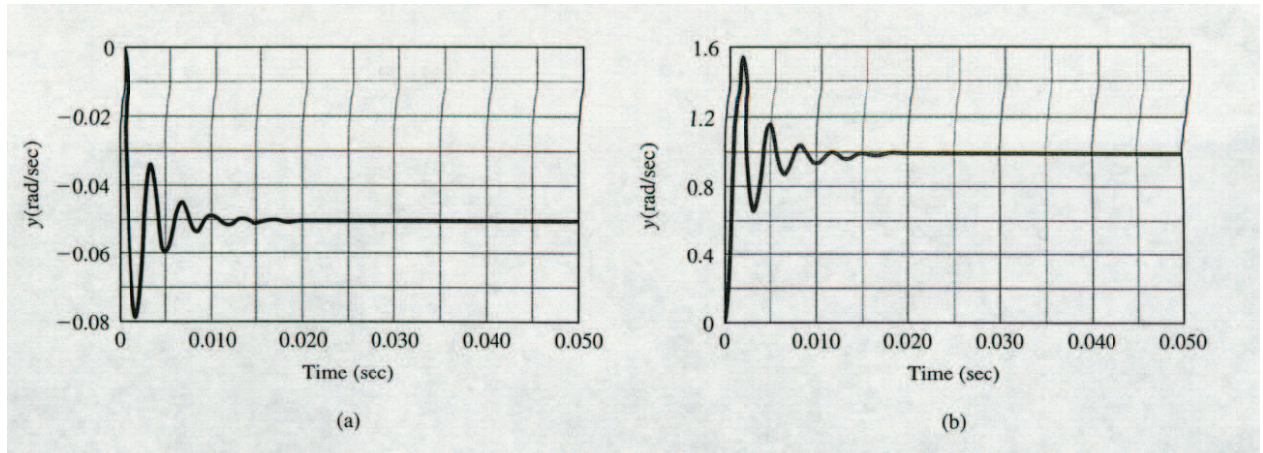


Figure 6: Closed-loop transient responses to (a) disturbance input and (b) reference input.

have that

$$y_{ss} = -\frac{5}{1 + AK} = -0.05. \quad (27)$$

The transfer function from the reference to the output is given by

$$T(s) = \frac{Y(s)}{R(s)} = \frac{10K}{\tau_1\tau_2s^2 + (\tau_1 + \tau_2)s + (1 + AK)}. \quad (28)$$

The system response to the given disturbance is shown in Figure 6(a), whereas the system response to a step reference input shown in Figure 6(b). Notice the differences in the responses of the two control systems!

Reading Assignment

See separate file on textbook reading assignments depending on the text edition you own. Read the examples in Handout E.19 posted on the course web page.