Texas A & M University
Department of Mechanical Engineering
MEEN 364 Dynamic Systems and Controls
Dr. Alexander G. Parlos

Lecture 20: The Classical Three-term PID Controller

The objective of this lecture is to present the elements of the most widely used control algorithm, that is the proportional (P), integral (I) and derivative (D) controller.

Proportional-Integral-Derivative Feedback Controllers

Proportional (P) Feedback Control

When the control signal is made proportional to the error in the measured output, we call the control law **proportional feedback** or P-control. Specifically, if

$$u(t) = Ke(t), \tag{1}$$

where K is the P-control gain. The controller transfer function $D(s)$ is

$$D(s) = K. \tag{2}$$

As we saw in the case of the DC motor control, unless the control gain is large, there will be steady-state error in the measured output. However, as we shall see later, as the control gain is made larger, the feedback loop stability becomes a serious issue. So, a P-controller has some fundamental limitations regarding its largest allowed gain in order to obtain well-damped response. Of course, such a gain limit might result in unacceptable steady-state error.

Proportional-Integral (PI) Feedback Control

The main reason for adding integral action is to eliminate any steady-state errors that might appear in the system response. Integral feedback has the form of

$$u(t) = \frac{K}{T_I} \int_{t_0}^{t} e(\eta) d\eta, \tag{3}$$

where T_I is called the **integral reset time**. The I-control transfer function $D(s)$ is

$$D(s) = \frac{K}{T_I s}. \tag{4}$$

To see the effect of I-control on the steady-state error, take the derivative of equation (3). This results in

$$\frac{du(t)}{dt} = \frac{K}{T_I} e(t), \tag{5}$$

which means that the control effort, $u(t)$, of an I-controller will stop varying, and take a non-zero value, only if the error $e(t)$ becomes zero. So, using I-control we can drive the steady-state error of $e(t)$ to zero.

The combination of P and I control, so-called PI-control can be expressed as

$$u(t) = K(e(t) + \frac{1}{T_I} \int_{t_0}^{t} e(\eta) d\eta), \tag{6}$$

where the proportional gain K and the integral reset time T_I are tunable parameters.

In general, I-control improves the steady-state response of a control system, but it also slows down its response. That is, unless we are willing to increase the overshoot in an attempt to improve the system's response speed.

Derivative (D) Feedback Control

Derivative or D-feedback has the form

$$u(t) = K T_D \frac{de(t)}{dt}, \tag{7}$$

with the corresponding transfer function being

$$D(s) = K T_D s, \tag{8}$$

where T_D is called the **derivative time**. This form of control is used to increase damping and improve system stability. It is usually used in conjunction with P and/or I-control because D-control by itself may not be effective if the control error, $e(t)$, is constant. D-control is anticipatory in nature, leading a P-only control by T_D seconds.

D-control has additional limitations, such as implementation problems and problems associated with the existence of sensor noise.

Proportional-Integral-Derivative (PID) Control

For more effective control over the steady-state and transient errors we can combine all three elements discussed before to obtain **proportional-integral-derivative (PID) control**. Some form of PID control (including P or PI or PD only) is used in probably over 90% or 95% of the industrial control loops.

The resulting PID control law is

$$u(t) = K(e(t) + \frac{1}{T_I}\int_{t_0}^{t} e(\eta)d\eta + T_D\frac{de(t)}{dt}), \tag{9}$$

with the corresponding transfer function given by

$$D(s) = K(1 + \frac{1}{T_I s} + T_D s). \tag{10}$$

The design of PID controllers deals with finding the parameters K, T_I and T_D such that design specifications are met. This controller parameter adjustment process is called **controller tuning**.

Example: Impact of PID Gains

We can see the effects of PID controller elements by considering the DC motor control problem we had studied before. First look at the disturbance rejection impact, see Figure 1(a). P-control only shows a large impact of the output as a result of a step disturbance input. As I-control is added

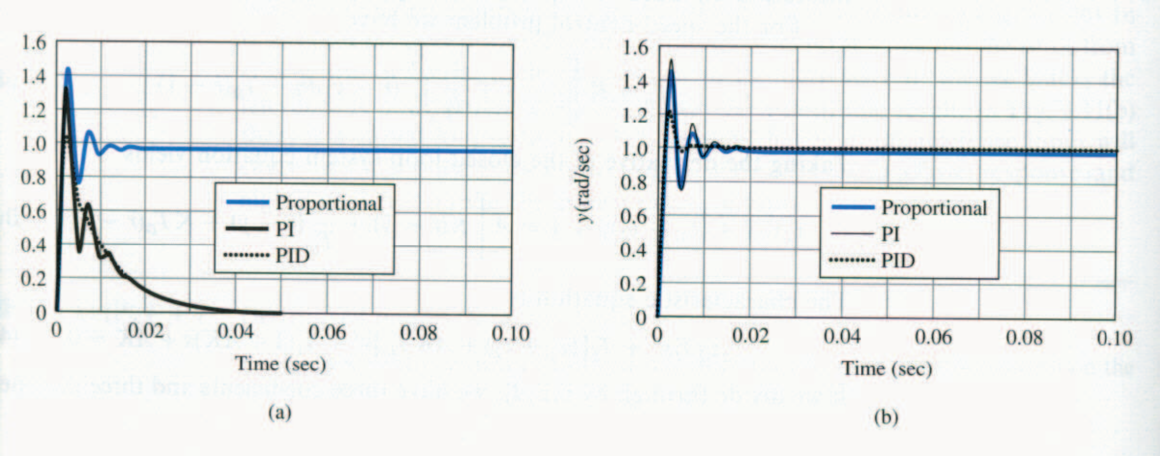

Figure 1: PID Control of DC Motor (a) step disturbance input; (b) step reference input.

the PI-controller results in zero steady-state error for a step input in disturbance. Further addition of the D-element, resulting in PID-control, results in a better-behaved response.

A similar behavior is observed for a step input change in the reference, see Figure 1(b). P-control only results in some steady-state error. Addition of the I-control removes the steady-state error, whereas addition of D-control results in a better-damped better-behaved response.

Reading Assignment

See separate file on textbook reading assignments depending on the text edition you own. Read the examples in Handout E.21 posted on the course web page.