Texas A & M University
Department of Mechanical Engineering
MEEN 364 Dynamic Systems and Controls
Dr. Alexander G. Parlos

Lecture 24: Dynamic Compensation Using Root-Locus

In addition to Bode plots, we can use the root-locus method to design dynamic compensators. The simplest form of a dynamic compensator takes the structure

$$D(s) = \frac{s+z}{s+p}, \tag{1}$$

where if $z < p$ it is a lead compensator, and if $z > p$ it is a lag compensator.

Lead Compensation

To understand the stabilizing effect of a lead compensator, we first consider $D(s) = s + z$. This is a PD controller and we apply it to the following second order system

$$KG(s) = \frac{K}{s(s+1)}. \tag{2}$$

The uncompensated and compensated root-locus is shown in Figure 1. The effect of a zero is to move the locus towards the stable region of the s-plane. Whereas before compensation achieving ω_n of, say, 2 would have resulted in very low damping (and high overshoot), following compensation we can achieve the same ω_n with damping ratio of more than 0.5.

The problem with the a pure lead compensator (zero only) is that its implementation requires use of a differentiator which is very sensitive to sensor noise. Furthermore, it is impossible to build a pure differentiator. However, the addition of a fast pole would not greatly reduce the effect of the zero. So, for example, we could suggest the following lead compensator,

$$D(s) = \frac{s+2}{s+20}. \tag{3}$$

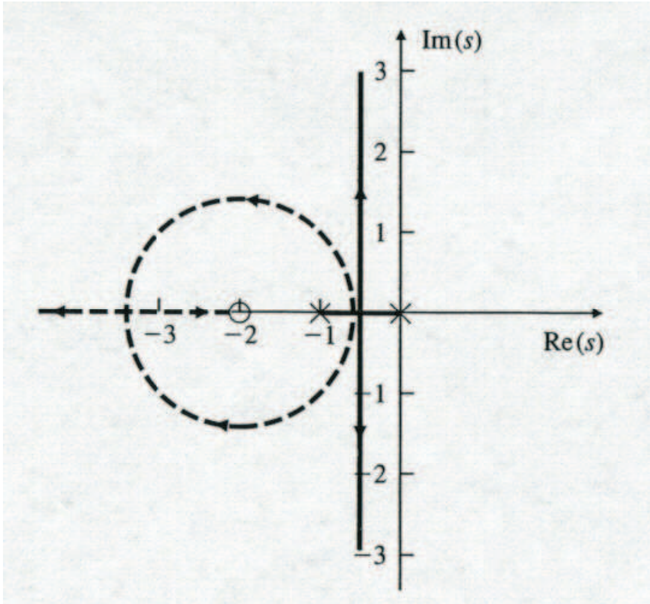

Figure 1: Root locus without compensation (solid line) and with PD compensation (dashed line).

The effect of the pole on the compensation can be seen in Figure 2.

Selecting exact values of z and p is usually done by trial and error. Generally, the zero is placed near the closed-loop pole. The choice of the compensator pole is a compromise between noise suppression and compensation effectiveness. The process can be made more analytical in nature, if the closed-loop pole is selected first. Then we arbitrarily select one of the lead-compensator parameters and use the angle criterion to select the other.

Lag Compensation

Once the desired transient response is obtained, one might discover that the steady-state response of the feedback loop is not satisfactory. Improvements in the steady-state errors can be made by placing a pole near the origin, which is usually accompanied by a zero nearby so that the pole-zero pair does not significantly interfere with the overall dynamic system response, shaped by the lead compensator.

For the problem studied previously, the lead compensator $\frac{s+2}{s+20}$ could be

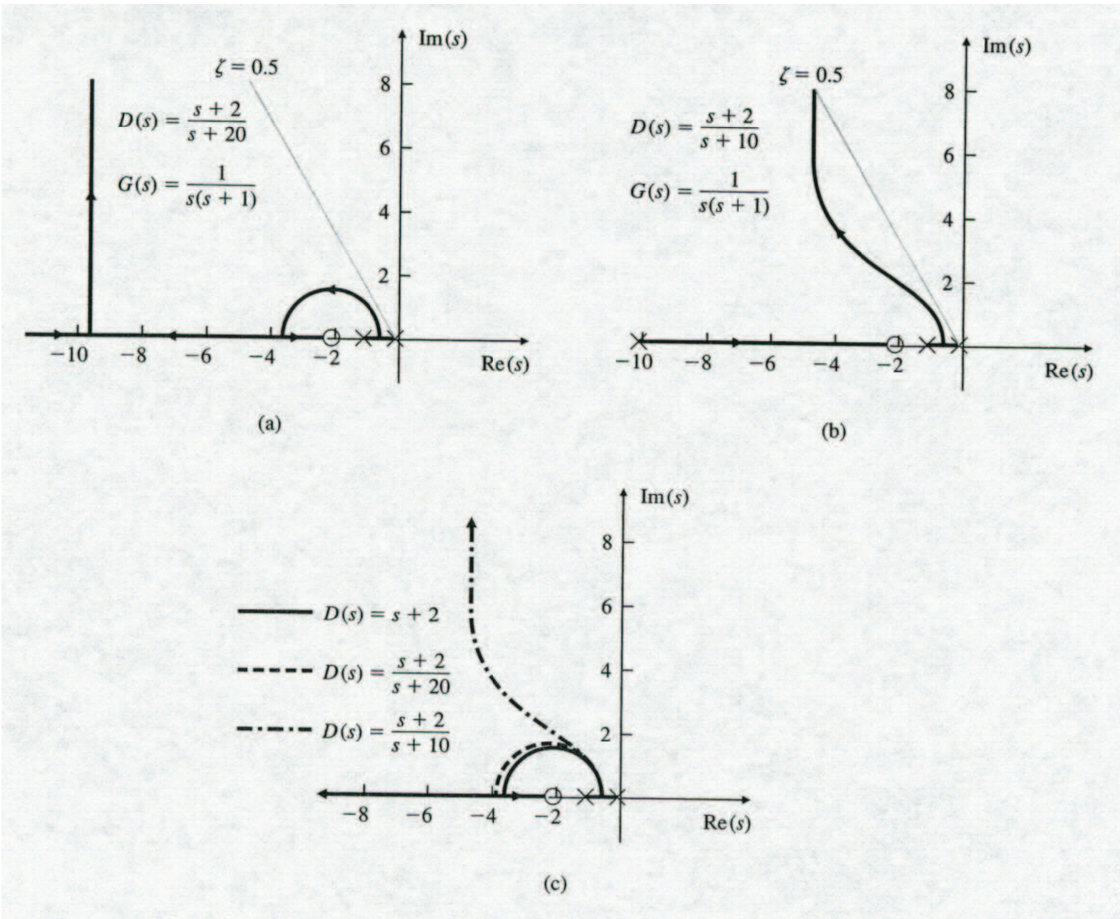

Figure 2: Effect of pole on compensation.

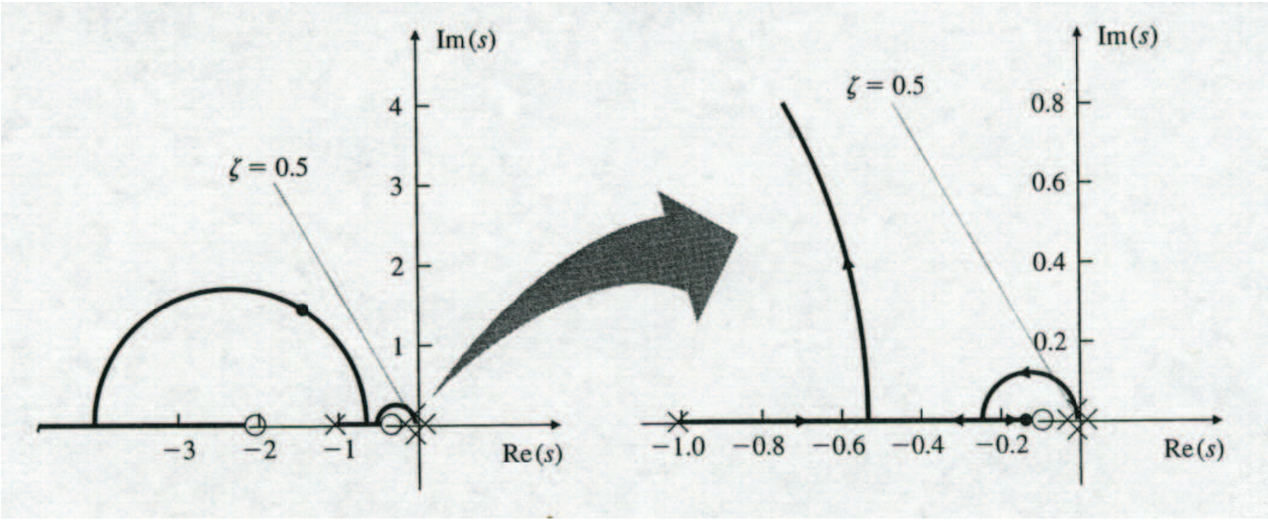

Figure 3: Effects of lag compensation on root locus.

followed by a lag compensator $\frac{s+0.1}{s+0.01}$. The lag will not impact the faster dynamics. However, a root close to the imaginary axis will persist and it will slow down the overall response. As a result, it is important to place the lag pole-zero at as high frequency as possible, without impacting the dominant system dynamics. The effect of the lag compensation on the root locus is shown in Figure 3.

Reading Assignment

See separate file on textbook reading assignments depending on the text edition you own. Read the examples in Handout E.28 posted on the course web page.