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MEEN 364 Dynamic Systems and Controls
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Lecture 25: Stability Margins and Compensation

Stability Margins

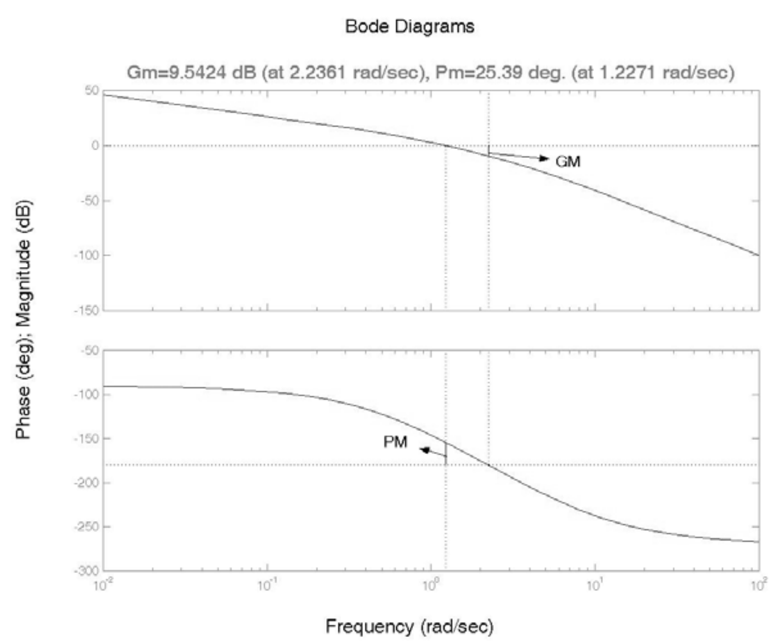
Two quantities that measure the stability margin of a system are the gain margin and phase margin.

Gain margin (GM) is the factor by which the forward loop system gain, i.e. the gain of $KG(j\omega)$, is less than the neutral stability value. For the typical case it can be read directly from the Bode plot by measuring the vertical distance between the $|KG(j\omega)|$ curve and the $|KG(j\omega)| = 1$ line at the frequency where $\angle G(j\omega) = 180^0$.

Phase margin (PM) is defined as the amount by which the phase of $KG(j\omega)$ exceeds -180° when $|KG(j\omega)| = 1$. The frequency at which the gain is unity is usually referred to as the **crossover frequency**.

It is relatively easy to determine these stability margins from the Bode plot. For example, consider the open loop system given by

$$G(s) = \frac{10}{s(s+1)(s+5)}.$$



Relation of the margins to other measures of performance

Consider the open loop system

$$G(s) = \frac{\omega^2}{s(s + 2\zeta\ \ \omega\ \ )}.$$

With unity feedback, the closed loop system is

$$T(s) = \frac{\omega^2}{s^2 + 2\zeta\ \ \omega\ \ s + \omega^2}.$$

The relation between the PM and the damping ratio of the system is given by

$$PM\ = \tan^{-1}\left(\frac{2\zeta}{\sqrt{\sqrt{1 + 4\zeta^4} - 2\zeta^2}}\right). \tag{1}$$

The above relation is depicted in Figure 1.

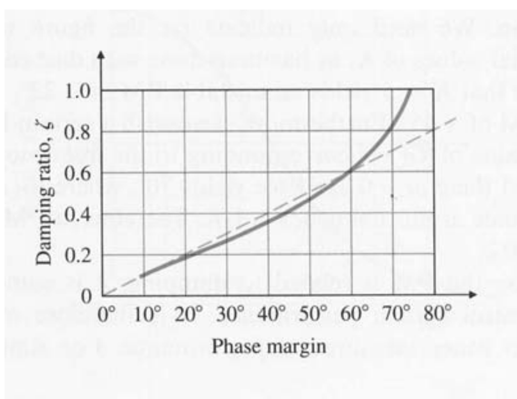


Figure 1. Damping ratio vs phase margin (PM)

Note that, the function is approximately a straight line up to about PM = 60°. The dashed line shows a straight line approximation to the function where

$$\zeta\ \ = \frac{PM}{100}. \tag{2}$$

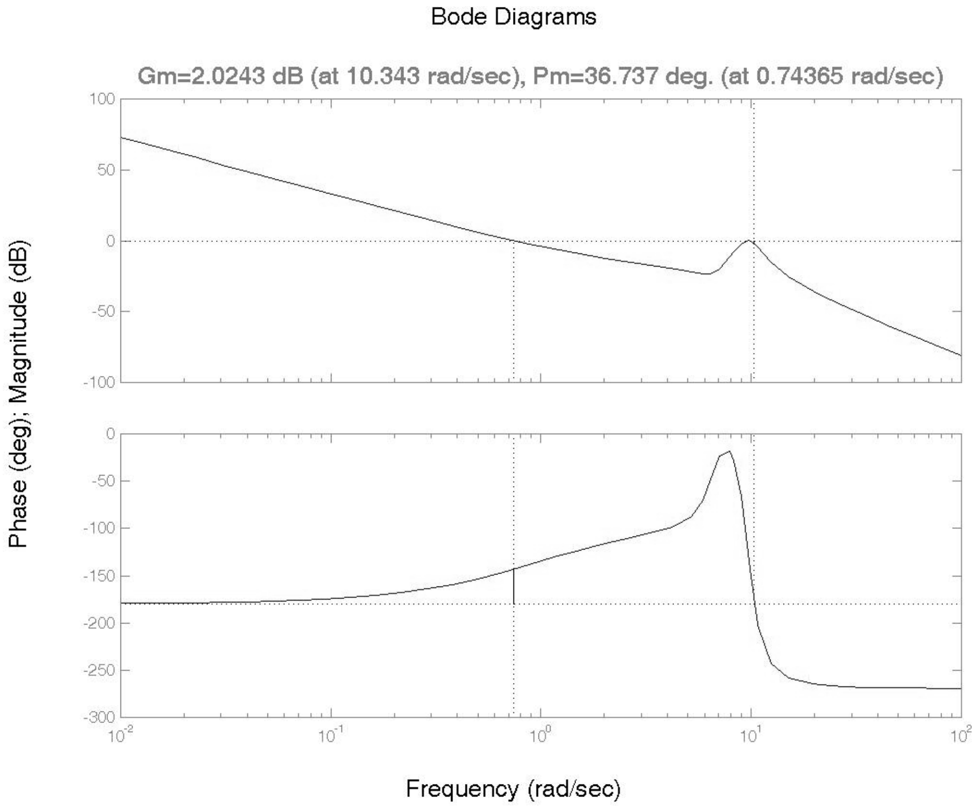
It is clear that the above approximation only holds for phase margins below about 70°. Furthermore, equation (1) is only accurate for the second order system. In spite of these limitations, equation (2) is often used as a rule of thumb for relating the damping ratio to PM. The gain margin (GM) for this second order system is infinite since the phase curve does not cross -180° as the frequency increases.

Example 1:

Consider the system defined by

$G(s) = \dfrac{85(s+1)(s^2+2s+43.25)}{s^2(s^2+2s+82)(s^2+2s+101)}$. Determine the stability margins.

From the bode plot it can be seen that, there are three crossings of the magnitude = 1 line, at 0.74, 9.5 and 9.8 rad/sec. The corresponding PM values are 37°, 43° and 71° respectively. We choose the minimum PM, as this is the most conservative assessment of the stability.



Closed-loop Frequency response

The bandwidth of the system is defined as the frequency at which the magnitude of the transfer function is equal to 0.707. This is depicted in Figure 2.

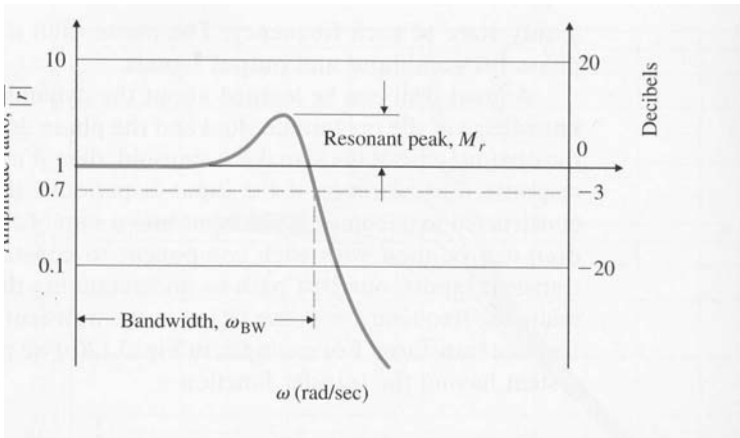


Figure 2. Definitions of bandwidth and resonant peak

The closed loop frequency response magnitude is approximated by

$$|T| = \left|\frac{KG(j\omega)}{1+KG(j\omega)}\right| \cong \begin{cases} 1, & \omega << \omega_c \\ |KG|, & \omega >> \omega_c \end{cases}.$$

Where ω_c is the crossover frequency. The above relation is shown in Figure 3.

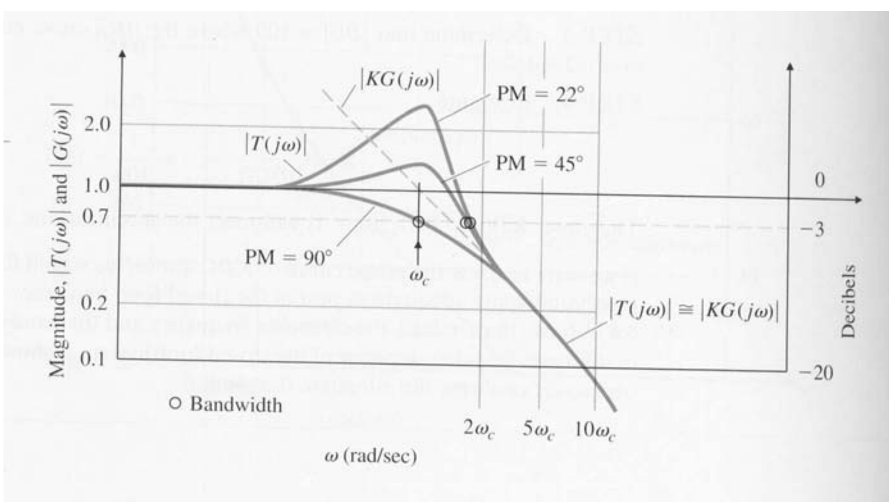


Figure 3. Closed-loop frequency response

Compensation

Dynamic elements (or compensation) are added to feedback control systems to improve their stability and error characteristics. The frequency response stability analysis to this point has considered the closed loop system to have the characteristic equation $1+KG(s)=0$. With the introduction of compensation, the closed loop characteristic equation becomes $1+KD(s)G(s)=0$ and all the previous discussion pertaining to the frequency response of KG(s) applies directly to the compensated case if we analyze the frequency response of KD(s)G(s).

PD Compensation

The compensator transfer function is given by

$$D(s)=K(T_D s+1). \quad (3)$$

The frequency response characteristics of equation (3) are shown in Figure 4. A stabilizing influence is apparent in the increase in phase at frequencies above the break point $1/T_D$. We use this compensation by locating $1/T_D$ so that the increased phase occurs in the vicinity of crossover thus increasing the phase margin.

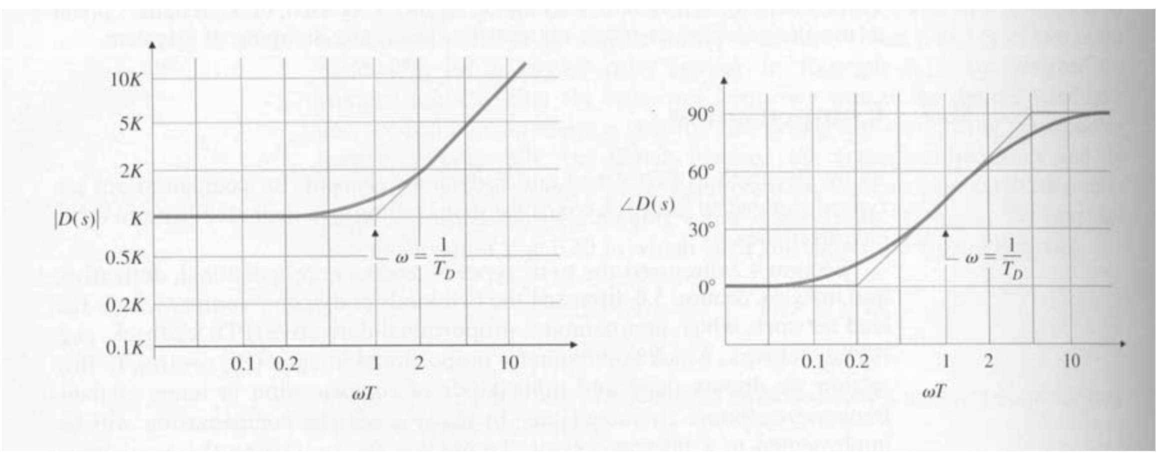


Figure 4. Frequency response of PD control

Note that the magnitude of the compensation continues to grow with increasing frequency. This feature is undesirable since it amplifies the high frequency noise that is typically present in any real system.

Reading Assignment

See separate file on textbook reading assignments depending on the text edition you own..