Texas A & M University
Department of Mechanical Engineering
MEEN 364 Dynamic Systems and Controls
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Lecture 5: Electrical and Electromagnetic System Components

The objective of this lecture is to review the fundamental components of electric and electromagnetic circuits. The laws governing electric circuits will be presented.

Basic Laws of Circuits

In electric circuits we talk about two-terminal elements, such as resistors, capacitors, etc., as shown in Figure 1. The electric potential at each terminal is measured by its voltage with respect to the ground or some other local reference potential, such as a machine frame or chassis. The rate of flow of electrical charge through the element, its current, is measured in terms of amperes or A. The fundamental equation relating these two quantities usually takes the form

$$e_{12}(t) = f_1(i_A(t)) \; or \; i_A(t) = f_2(e_{12}(t)). \tag{1}$$

In addition to the two-terminal elements, there are two types of ideal sources used to drive circuits, as shown in Figure 2. The ideal voltage source, capable of delivering designated voltage level e_s regardless of the current drawn, and the ideal current source, capable of delivering the designated current i_s regardless of the voltage required to drive the load.

Two basic laws govern the operation of circuits. These are known as Kirchoff's voltage law and Kirchoff's current law. The voltage law says that the

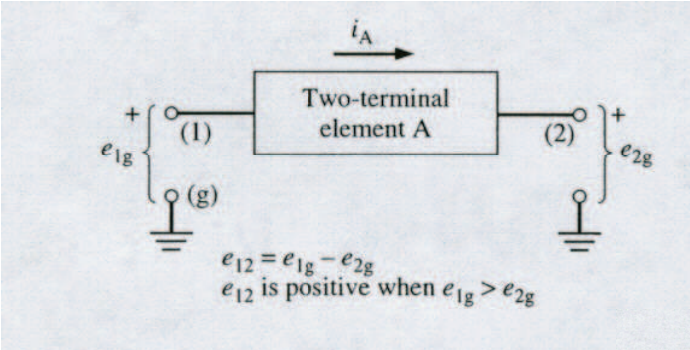

Figure 1: Circuit diagram of a two-terminal electrical element.

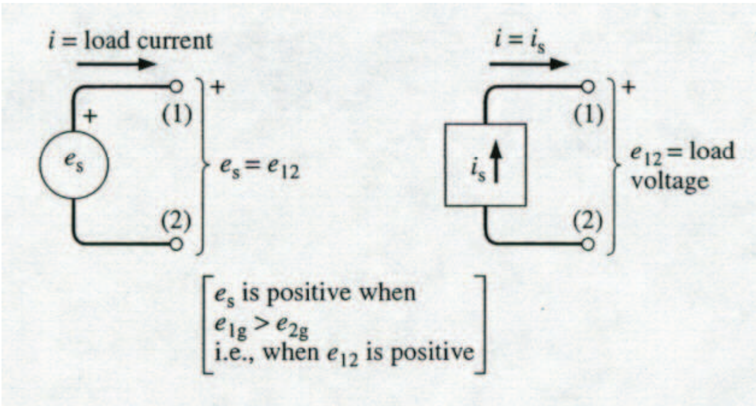

Figure 2: Circuit diagram of a voltage and current sources.

sum of the voltage drops around a loop must be zero. The current law says that the sum of the currents at a node (the junction of two or more elements) must be zero. These two laws are illustrated in Figures 3 and 4.

Capacitors

A capacitor is used to store electric charge. The equation describing the capacitor charge is

$$q_C(t) = Ce_{12}(t). \tag{2}$$

In terms of the current (the rate of change of the charge), the governing

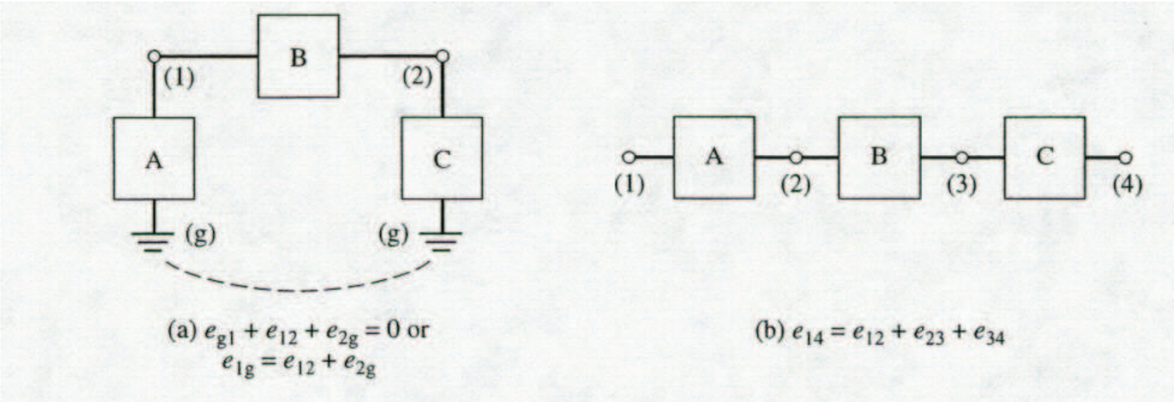

Figure 3: Kirchoff's Voltage Law.

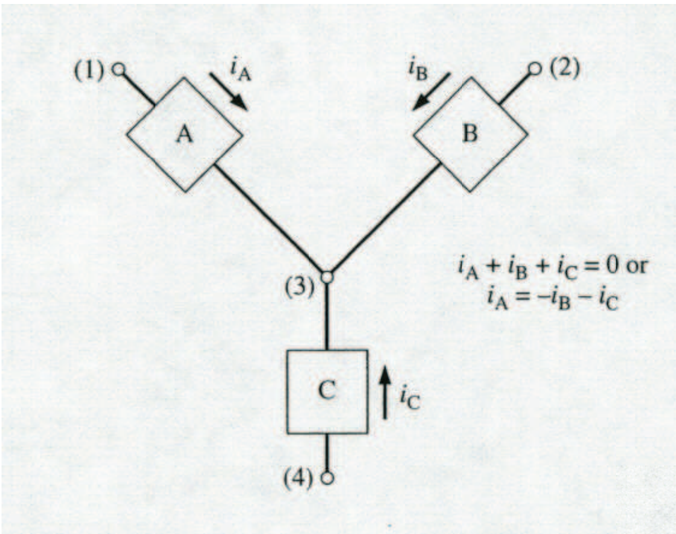

Figure 4: Kirchoff's Current Law.

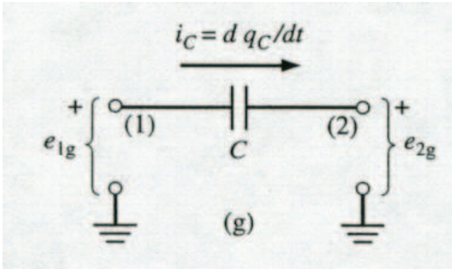

Figure 5: Circuit diagram of an ideal capacitor.

equation for a capacitor is

$$i_C(t) = C\frac{de_{12}(t)}{dt}. \tag{3}$$

The state variable of a capacitor is the its voltage $e_{12}(t)$.

A capacitor, like a mass m, is an energy storage element and it is used to store electrical energy. This energy is in the form of a static field and it can be expressed as

$$\boldsymbol{\mathcal{E}}_e(t) = \frac{C}{2}e_{12}^2(t). \tag{4}$$

As in the arguments about a mass m, attempts to suddenly change the voltage across a capacitor would require an infinite power source. However, one can suddenly change the capacitor current. An ideal capacitor is shown in Figure 5.

Inductors

The variable governing the operation of an inductor is the flux linkage, λ_{12}. It can be expressed in terms of the current flowing through the inductor as

$$\lambda_{12}(t) = Li_L(t). \tag{5}$$

In terms of the voltage, $e_{12}(t) = \frac{d\lambda_{12}(t)}{dt}$ and the governing equation for an inductor is expressed as

$$e_{12}(t) = L\frac{di_L(t)}{dt}. \tag{6}$$

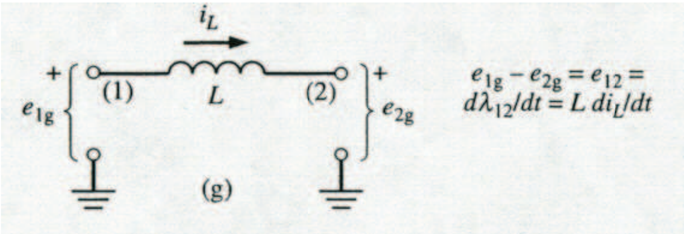

Figure 6: Circuit diagram of an ideal inductor.

Note the similarity of an inductor to the ideal spring. The state variable of an inductor is its current $i_L(t)$.

The energy stored in an inductor is in the magnetic field surrounding its conductors, and it is known as magnetic field energy. The stored magnetic field energy can be expressed as

$$\boldsymbol{\mathcal{E}}_m(t) = \frac{L}{2} i_L^2(t). \tag{7}$$

As in the arguments about a spring k, attempts to suddenly change the current flowing though an inductor would require an infinite power source. However, one can suddenly change the voltage across an inductor. An ideal inductor is shown in Figure 6.

Transformers

If two coils of wire are installed very close to each other so that they share the same core without flux leakage, an electric transformer results, as shown in Figure 7.

A transformer is a four-terminal element and two equations are needed to describe its operation. The equations describing the operation of a transformer are

$$e_{34}(t) = n e_{12}(t), \tag{8}$$

where n is the ratio of the number of turns between (3) and (4) to the number

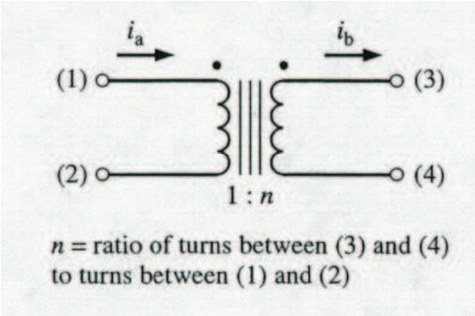

Figure 7: Circuit diagram of an ideal transformer.

of turns between (1) and (2), and

$$i_b(t) = \frac{1}{n} i_a(t). \tag{9}$$

Equation (8) and (9) indicate that transformers do not store energy, rather are used to couple circuits dynamically.

Resistors

The equation governing the operation of an ideal resistor is Ohm's law. It can be expressed as

$$e_{12}(t) = R i_R(t). \tag{10}$$

Note that the voltage across a resistor and the current through it are related "instantaneously" to each other. This is because there is no energy storage, rather dissipation.

A circuit diagram of an ideal resistor is shown in Figure 8.

Examples of Circuit Analysis

Example 1

Develop the input-output differential equation for the circuit shown in

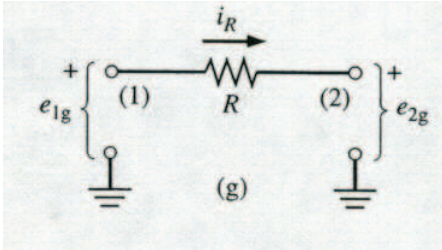

Figure 8: Circuit diagram of an ideal resistor.

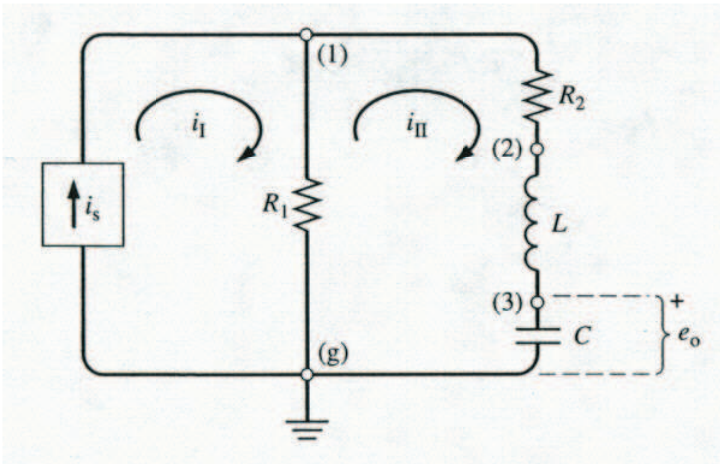

Figure 9: An R, L, C circuit driven by a current source.

Figure 9.

We use the so-called loop method to derive the equations describing the circuit operation. As shown in Figure 9 there are two independent loops in this circuit. We name the current flowing through these loops as i_I and i_{II}.

It is obvious that from loop I we can immediately write

$$i_I(t) = i_s(t). \tag{11}$$

For Loop II we can write Kirchoff's voltage law as follows:

$$R_2 i_{II}(t) + L\frac{di_{II}(t)}{dt} + \frac{1}{C}\int i_{II}(t)dt + R_1(i_{II}(t) - i_s(t)) = 0. \tag{12}$$

We can relate the capacitor voltage $e_0(t)$ with the capacitor current $i_{II}(t)$ as

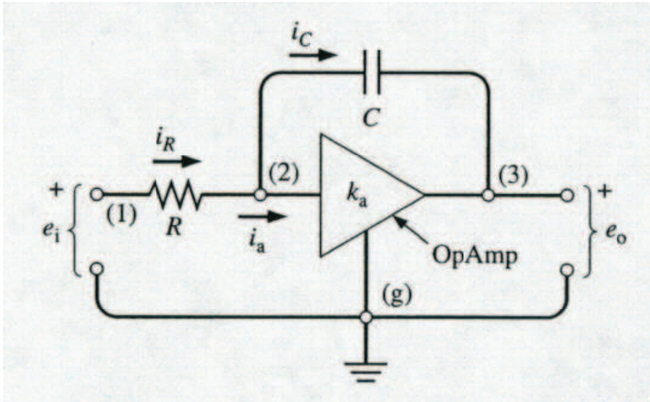

Figure 10: High-gain OpAmp with capacitor feedback.

follows

$$i_{II}(t) = C\frac{de_0(t)}{dt}. \tag{13}$$

As a result, equation (12) can be rewritten as

$$(R_1 + R_2)C\frac{de_0(t)}{dt} + LC\frac{d^2e_0(t)}{dt^2} + e_0(t) = R_1 i_s(t), \tag{14}$$

or by rearranging we have

$$LC\frac{d^2e_0(t)}{dt^2} + (R_1 + R_2)C\frac{de_0(t)}{dt} + e_0(t) = R_1 i_s(t). \tag{15}$$

Example 2

The circuit shown in Figure 10 involves the use of a high-gain operational amplifier (OpAmp) with feedback to achieve desired dynamic response in automatic controllers. The gain k_a of the OpAmp is negative, and its input current i_a is so small that it can be considered negligible. The objective is to develop an input-output model for this circuit.

We use Kirchoff's current law at node (2). This yields

$$i_R(t) = i_C(t), \tag{16}$$

because the current i_a is assumed negligible. Equation (16) can be rewritten as

$$\frac{e_i(t) - e_{2g}(t)}{R} = C\frac{d(e_{2g}(t) - e_{3g}(t)))}{dt}. \tag{17}$$

For the amplifier we have the following equation

$$e_{3g}(t) = k_a e_{2g}(t), \tag{18}$$

or

$$e_{2g}(t) = \frac{1}{k_a} e_{3g}(t). \tag{19}$$

Combining equations (17) and (19) results in

$$e_i(t) - \frac{1}{k_a} e_{3g}(t) = RC\frac{d[(\frac{e_{3g}(t)}{k_a}) - e_{3g}(t)]}{dt}. \tag{20}$$

Rearranging yields

$$RC\frac{[1 - (\frac{1}{k_a})]de_{3g}(t)}{dt} - \frac{1}{k_a} e_{3g}(t) = -e_i(t). \tag{21}$$

Considering that for an amplifier, k_a is very large, we have

$$\frac{de_{3g}(t)}{dt} \approx -\frac{e_i(t)}{RC}, \tag{22}$$

or

$$e_{3g}(t) = -(\frac{1}{RC}) \int_{0^-}^{t} e_i(t)dt + e_{3g}(0^-). \tag{23}$$

The use of a capacitor in the feedback with a resistor at the input results in an integrator with time constant RC, as shown in the block diagram of Figure 11.

Reading Assignment

See separate file on textbook reading assignments depending on the text edition you own. Read examples Handout E.8 posted on the course web page.

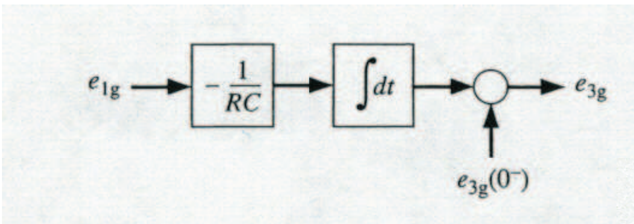

Figure 11: Simulation block diagram of an integrator with time constant.