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MEEN 364 Dynamic Systems and Controls
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Lecture 6: Modeling of Electromechanical Systems
– Principles of Motor Operation

The objective of this lecture is to give you some background on modeling systems with components in mixed energy domains, such as electromechanical systems and discuss the principles of their operations.

In many engineering applications we must combine elements from several different energy domains, such as electrical and mechanical domains. In order to combine these domains we need coupling devices that convert one kind of energy or signal to another, i.e. electrical to mechanical. The general term "transducer" will be used for ideal coupling devices. In cases where significant amounts of energy or power is involved, these will be called "energy-converting transducers". Otherwise, they will be referred to as "signal-converting transducers".

Energy-Converting Transducers

We start our discussion with ideal transducers, where no energy is stored or dissipated in the elements. Two types of such transducers are discussed.

Translational-to-Rotational Mechanical Transuders

A diagram of such a transducer is shown in Figure 1, where n is the coupling coefficient relating output motion to input motion.

The equations for this transducer are
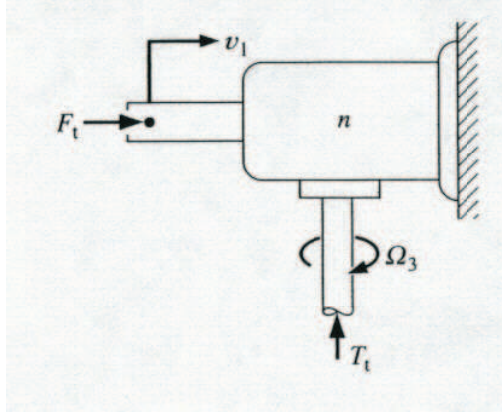
$$\Omega_3 = nv_1, \tag{1}$$

Figure 1: A symbolic diagram of an ideal translational-to-rotational mechanical transducer.

$$nT_t = F_t. \tag{2}$$

Why? Hint: Power in must equal power out.

Examples of such transducers include pulley-and-cable systems, level-and-shaft mechanisms, and rack-and-gear mechanisms.

Electromechanical Energy Converters

Again, we have two types of electromechanical transducers, as shown in Figure 2. Translational mechanical electromechanical transducers, such as solenoids and linear motors, and rotational mechanical electromechanical transducers, such as rotaty electric motors or electric generators. The same rotational device can operate as a motor and a generator depending on the direction of the power flow (or the sign of the input power).

Also given in Figure 2 are the equations governing the operation of such devices. Although these equations are intended primarily for use with direct current (DC) devices, they could be applied to alternative current (AC) devices as long as the root mean square (rms) values of the voltage and current are used.

Electric current and magnetic fields interact in many ways, however, three laws associated with these interactions are very important in understanding
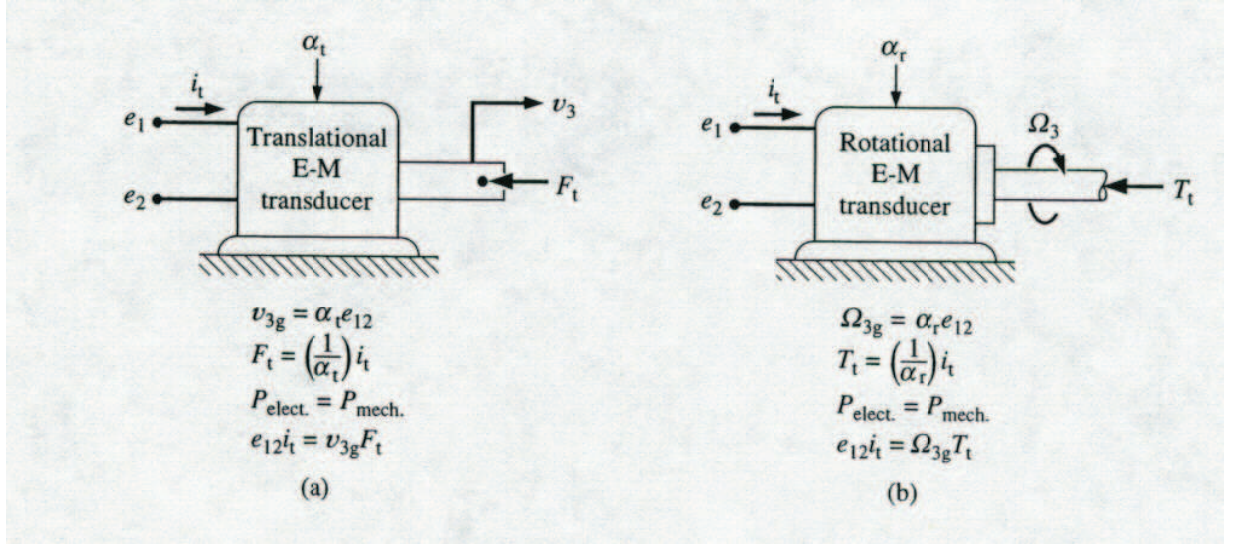
Figure 2: A symbolic diagram of an ideal electromechanical transducers: (a) translational mechanical and (b) rotational mechanical.

the operation of most electromechanical systems.

Magnetic Field Proportional to Current: The first relation states that an electric current establishes a magnetic field. The strength of the magnetic field at a given point depends on the intensity of the current, the materials, and the geometry involved. A typical case is shown in Figure 3. Here a toroid of radius R and of material with permeability μ is wrapped with N turns of wire carrying i amperes. The field strength at the center of the toroid is given by

$$B = \frac{\mu}{2\pi R} N i. \tag{3}$$

Law of Motors - Force is Proportional to Current and Magnetic Field: The second important law states that a charge q moving with a velocity $\mathbf{v}$ in a magnetic field with intensity $\mathbf{B}$ experiences a force $\mathbf{F}$ given by $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$. If the moving charge is composed of a current i amperes in a conductor of lenght l meters arranged in at right angle to a field of strength B tesla, then the force is at right angles to the plane of i and B, and has the magnitude
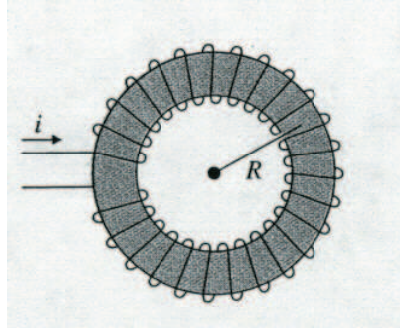
$$F = B l i \; Newtons. \tag{4}$$

Figure 3: Toroid wrapped with N turns of wire.

This equation forms the basis for converting electrical energy to mechanical energy and it is called the law of motors.

Law of Generators - Voltage is Proportional to Velocity and Magnetic Field: The last important law relates conversion of mechanical energy to electrical energy and it is the opposite of the previous law. It states that if a charge is moving in a magnetic field and is forced along a conductor, an electric voltage is established between the ends of the conductor. If a conductor of length l meters is moved at a velocity v meters per second through a constant field of B tesla at right angles to the direction of the field, then the voltage between the ends of the conductor is given by

$$e = Blv \ Volts. \tag{5}$$

This equation forms the basis for converting mechanical energy to electrical energy and it is called the law of generators.

DC motor actuators: A common actuator used in control systems is the DC motor. It provides rotary motion and it can also provide translational motion. A sketch of a DC motor is given in Figure 4. The components of the motor are: (1) housings and bearings, (2) stator (magnets; permanent or electromagnets), (3) rotor, (4) commutator and (5) brushes.

The brushes force current through the wire wound around the rotor. The rotating commutator causes current always to be sent through the armature, a collection of conductor windings, so that it will produce the maximum
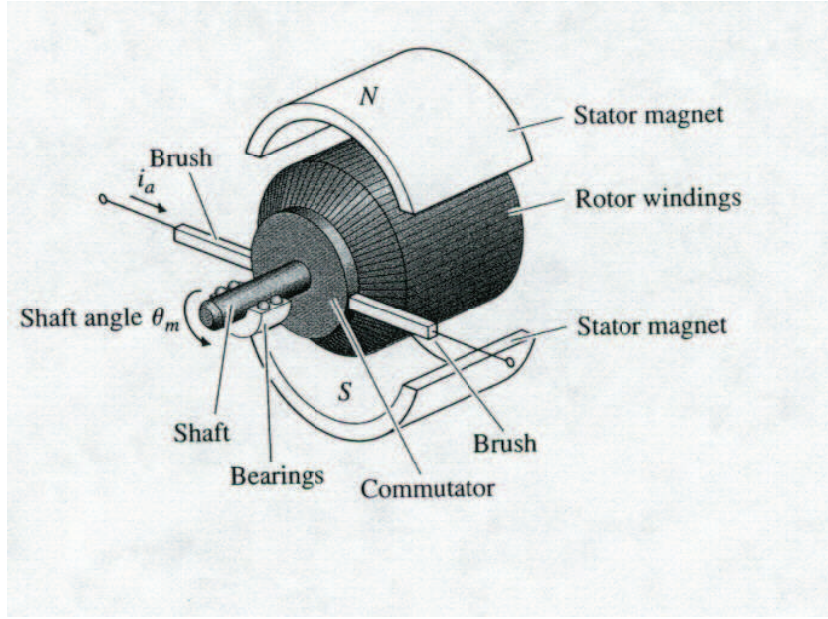
Figure 4: DC motor sketch.

torque in the desired direction. If the armature current direction is reversed, the torque direction is reversed also. The operation of a DC motor is usually expressed by the equation relating the torque developed in the rotor in terms of the armature current, i_a, and the voltage generated (called back electromotive force or emf) as a result of rotation in terms of the shaft's rotational velocity, $\dot{\theta}_m$. Thus,

$$T = K_t i_a, \tag{6}$$

$$e = K_e \dot{\theta}_m. \tag{7}$$

In consistent units, $K_t = K_e$.

AC motor actuators: Another device used for electromechanical energy conversion is the AC induction motor. Suppose we construct a rotor of conductors with no commutator or brushes. Now suppose that the stator magnetic field can be made to rotate at high speed, say 1800 *rpm*. The moving field will *induce* currents in the rotor, which will in turn be subject to forces according to the motor law. These forces will cause the rotor to turn so that it follows the rotating magnetic field. This is a simplified principle of
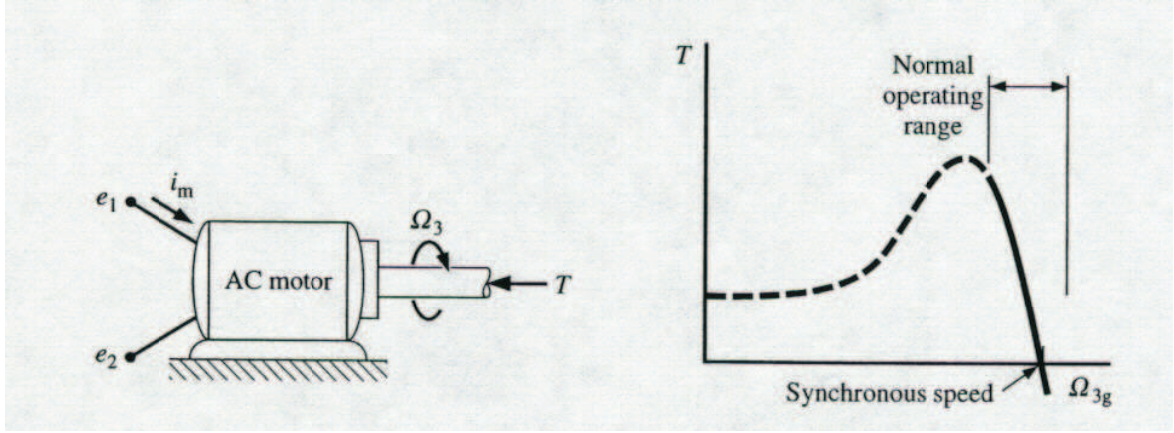
Figure 5: Steady-state torque-speed curve for squirrel-cage AC induction motor operating at constant supply frequency and voltage.

operation of AC induction motors.

AC induction motor of the squirrel-cage type operate well only at speeds close to their no-load speed. A typical torque-versus-speed curve of such a motor operating a constant supply voltage is shown in Figure 5. This curve accounts for friction effects and other losses. Thus it is necessary to know the motor efficiency in order to determine the motor current. Given the motor efficiency of a single phase AC induction motor, η_m, the motor current rms value is given by

$$i_{m_{rms}} = \frac{1}{\eta_m} \frac{T\dot{\theta}_3}{e_{12_{rms}}PF}. \tag{8}$$

Signal-Converting Transducers

These devices could be energy-converting transducers with negligible load or specially designed devices that convert one type of signal to another. Examples of such devices include tachometer generator, linear velocity sensor, piezo-electric force sensor, etc.
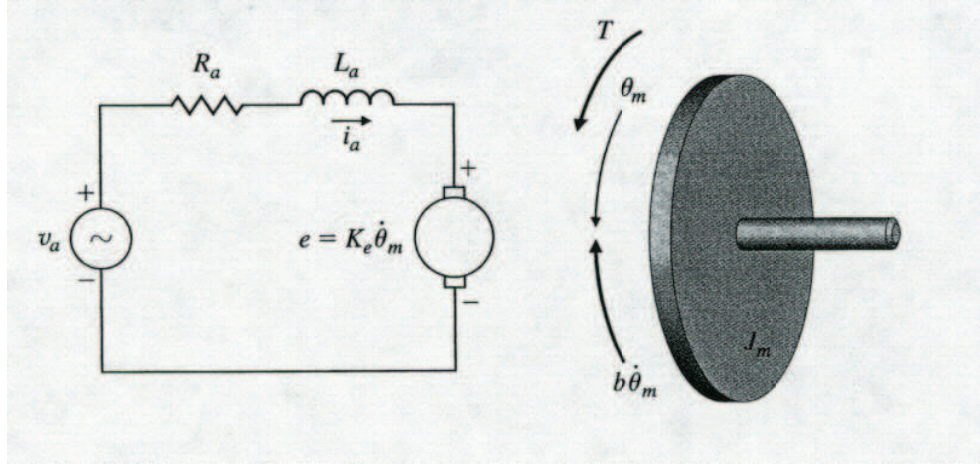
Examples of Mixed Systems

Figure 6: DC motor: (a) electric circuit of the armature; (b) free-body diagram of the rotor.

Example 1 - Modeling of DC Motors

Find a set of equations for the DC motor with the armature driven by the electric circuit shown in Figure 6(a). Assume the rotor has inertia J_m and friction coefficient b.

In the electrical part of the system we consider the back emf of the electrical circuit. In the mechanical part of the system we need to include the motor torque developed. The free-body diagram of the rotor is shown in Figure 6(b). If we apply the law of conservation of angular momentum we have

$$J_m\ddot{\theta}_m(t) = T_m(t) + T_{load}(t). \tag{9}$$

The motor torque is given by

$$T_m(t) = K_t i_a(t), \tag{10}$$

and the load in this case is the rotor inertia. The only torque generated by the load is the friction (or damping) torque expressed as

$$T_{load}(t) = -b\dot{\theta}_m(t). \tag{11}$$

Applying Kirchoff's voltage law on the electrical system we have

$$L_a\frac{di_a(t)}{dt} + R_a i_a(t) = v_a(t) - K_e\dot{\theta}_m(t). \tag{12}$$

If we define the state vector as

$$\mathbf{x}(t) = [\theta_m(t)\dot{\theta}_m(t)i_a(t)]^T, \tag{13}$$

and the input as $u(t) = v_a(t)$, we obtain the following state-space representation of a DC motor model

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t), \tag{14}$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{b}{J_m} & \frac{K_t}{J_m} \\ 0 & -\frac{K_e}{L_a} & -\frac{R_a}{L_a} \end{bmatrix}, \tag{15}$$

and

$$\mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L_a} \end{bmatrix}. \tag{16}$$

Note in most instances the electrical part of the system will be much faster than the mechanical component. As a result an applied voltage will result in an almost instantaneous current flow. Therefore, it is often feasible to neglect the inductance L_a in equation (12). The resulting equation is now of second order

$$J_m\ddot{\theta}_m(t) + (b + \frac{K_tK_e}{R_a})\dot{\theta}_m(t) = \frac{K_t}{R_a}v_a(t). \tag{17}$$

Equation (17) indicates that the mechanical friction and the back emf are indistinguishable. They both act to damp the motion of the DC motor.

Reading Assignment

See separate file on textbook reading assignments depending on the text edition you own. Read examples Handout E.9 posted on the course web page.