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MEEN 364 Dynamic Systems and Controls
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Lecture 7: Thermal and Fluidic Systems

Basic Mechanisms of Heat Transfer

Energy may be transferred across the boundaries of a system, either to or from the system. It occurs only when there is a temperature difference between the system and the surroundings. Energy is transferred by *conduction*, *convection* and *radiation*, which may occur separately or in combination. We start our discussion by introducing the basic physical mechanisms by which thermal energy is transferred.

Conduction

Conduction heat transfer occurs only when there is physical contact between bodies (systems) at different temperatures. It can also be defined as the transfer of energy through a substance resulting from a difference in temperature in the different parts of the substance. For one-dimensional heat conduction in the x direction, the rate of heat flow is determined by the Fourier equation

$$Q_{hk} = -kA\frac{dT}{dx}, \tag{1}$$

where A is an area of heat transfer normal to x and k is the thermal conductivity, which is defined as the heat flow per unit area per unit time when the temperature decreases by one degree in a unit distance. For example, one-dimensional steady state heat conduction is depicted in Figure 1.

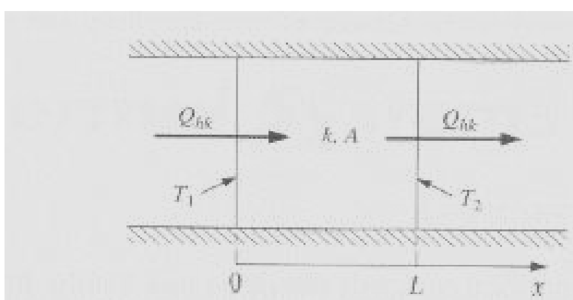

Figure 1: One-dimensional steady state heat conduction

From equation (1), the rate of heat flow can be written as

$$Q_{hk} = -kA\frac{dT}{dx}.$$

Integrating the above equation with respect to *x*, we obtain

$$\int_0^L Q_{hk}\,dx = \int_{T_1}^{T_2} -kA\,dT,$$

$$\Rightarrow Q_{hk} = -\left(\frac{A}{L}\right)\int_{T_1}^{T_2} k\,dT.$$

If the thermal conductivity of the material, ‘k’ does not depend on temperature, the rate of heat transfer can be expressed as

$$Q_{hk} = \left(\frac{kA}{L}\right)(T_1 - T_2).$$

Convection

Convective heat transfer is defined as the heat transfer between a fluid and a solid when there is temperature difference between the solid and the fluid. It is usually associated with the significant motion of the fluid around the solid. The rate of heat transfer by convection between a solid and a fluid flowing around it is given by

$$Q_{hc} = h_c A(T_s - T_f),$$

where h_c is the convective heat transfer coefficient, A is the area of heat transfer and T_s and T_f represent the solid and fluid temperatures respectively.

Radiation

This is the means by which heat is transferred for example from the sun to the earth through mostly empty space. The rate of heat transfer by radiation between two separated bodies having temperatures T_1 and T_2 is determined by the Stefan-Boltzmann law,

$$Q_{hr} = \sigma\ F_E F_A A(T_1^4 - T_2^4),$$

where

$\sigma = 5.667 \times 10^{-8}$ W/m^{2}k^4 (The Stefan-Boltzmann constant)
F_E is the effective emissivity
F_A is the shape factor
A is the heat transfer area.

The effective emissivity accounts for the deviation of the radiating systems from black bodies. The values of the shape factor range from 0 to 1 and represent the fraction of the radiative energy emitted by one body that reaches the other body.

Lumped Models of Thermal Systems

Mathematical models of thermal systems are usually derived from the basic energy balance equations that follow the general form

$$\begin{pmatrix} \text{rate of energy} \\ \text{stored} \\ \text{within the system} \end{pmatrix} = \begin{pmatrix} \text{heat flow} \\ \text{rate} \\ \text{in to system} \end{pmatrix} - \begin{pmatrix} \text{heat flow} \\ \text{rate} \\ \text{out of system} \end{pmatrix} + \begin{pmatrix} \text{rate of heat} \\ \text{generated} \\ \text{within system} \end{pmatrix} + \begin{pmatrix} \text{rate of work} \\ \text{done} \\ \text{upon system} \end{pmatrix}$$

Fluid System Elements

Fluid Capacitors

A fluid capacitor is shown in Figure 2.

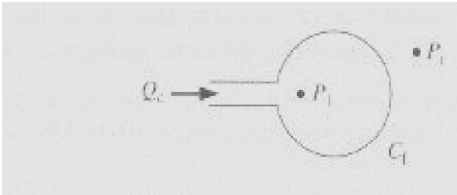

Figure 2: Symbolic diagram of a fluid capacitor.

The pressure in a fluid capacitor must be referred to a reference pressure P_r. When the reference pressure is that of the surrounding atmosphere, it is the gage pressure. When the reference pressure is zero, i.e., a perfect vacuum, it is the absolute pressure. The volume flow rate Q_c is given by

$Q_c = C_f \dfrac{dP_{1r}}{dt}$, where C_f is the fluid capacitance.

The net flow into the capacitor is stored and corresponds somewhat to the energy stored in the charging process. The potential energy stored in an ideal fluid capacitor is given by

$E_P = \dfrac{C_f}{2} P_{1r}^2$.

Fluid Inertors

The symbolic diagram of a fluid inertor is shown in Figure 3.

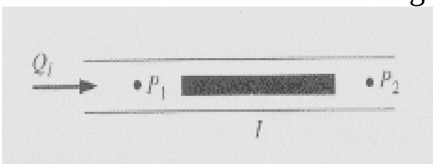

Figure 3: A symbolic diagram of a Fluid inertor.

The elemental equation for the inertor is

$P_{12} = I\frac{dQ_I}{dt}$, where I is the fluid inertance. For frictionless incompressible flow in a uniform passage having cross sectional area A and length L, the inertance is $I = \frac{\rho\ L}{A}$, where ρ is the mass density of the fluid.

The kinetic energy stored in an ideal inertor is given by

$E_K = \frac{1}{2}Q_i^2$.

Fluid Resistors

The symbolic diagram of a fluid resistor is shown in Figure 4.

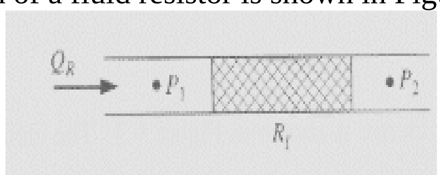

Figure 4: A symbolic diagram of a fluid resistor.

The elemental equation of an ideal resistor is

$P_{12} = R_f Q_R$.

Fluid Sources

The ideal sources employed in fluid system analysis are shown in Figure 5. An ideal pressure source is capable of delivering the indicated pressure, regardless of the flow required by what it is driving, whereas an ideal flow source is capable of delivering the indicated flow rate, regardless of the pressure required to drive its load.

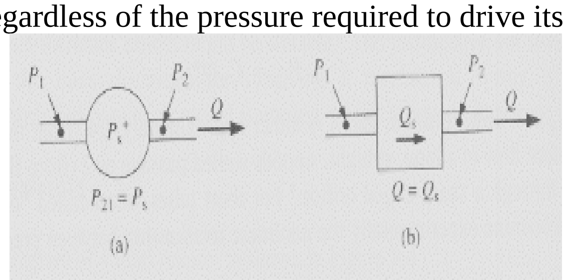

Figure 5: Ideal fluid sources a) pressure source and b) flow source

Interconnection Laws

The two fluid system interconnection laws are the law of continuity and compatibility. The continuity law says that the sum of the flow rates at a junction must be zero, and the compatibility law says that the sum of the pressure drops around a loop must be zero. The laws are illustrated in Figure 6.

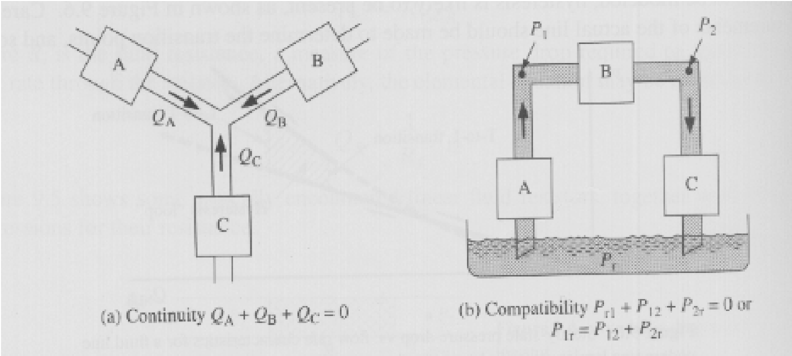

Figure 6: Interconnection laws

The continuity law states that

$$Q_A + Q_B + Q_C = 0.$$

The Compatibility law states that

$$P_{r1} + P_{12} + P_{2r} = 0,$$
$$\Rightarrow P_{1r} = P_{12} + P_{2r}.$$

Example 1 – Modeling of a Heat Exchanger

A heat exchanger is shown in Figure 7. Steam enters the chamber through the controllable valve at the top, and the cooler steam leaves at the bottom. There is a constant flow of water through the pipe that winds through the middle of the chamber so that it picks up thermal energy from the steam. Find the differential equations that describe the dynamics of the measured water outflow temperature as a function of the area.

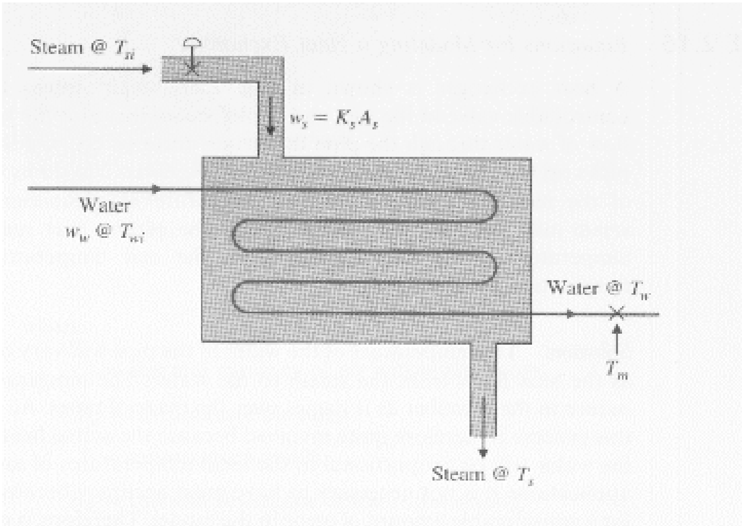

Figure 7: Heat Exchanger

Let T_s and T_w be the temperatures for the outflow steam and the water respectively. The heat transfer from the steam to water is proportional to the difference in these temperatures. In other words, the heat transfer is through convection, hence

$$q_c = hA(T_s - T_w).$$

The flow of thermal energy into the chamber from the inlet steam depends on the steam flow rate and its temperature according to

$$q_{in} = w_s c_{vs}(T_{si} - T_s),$$

where

$w_s = K_s A_s$, mass flow rate of the steam,
A_s = area of the steam inlet valve,
K_s = flow coefficient of the inlet valve,
c_{vs} = specific heat of the steam,
T_{si} = temperature of the inflow steam,
T_s = temperature of the outflow steam.

Therefore the net flow rate into the chamber is the difference between the heat from the hot incoming steam and the thermal energy flowing to the water. This net flow determines the rate of temperature change of the steam according to

$$C_s \dot{T}_s = A_s K_s c_{vs}(T_{si} - T_s) - hA(T_s - T_w), \tag{2}$$

where

$C_s = m_s c_{vs}$ is the thermal capacity of the steam in the chamber with mass m_s.

Likewise, the differential equation describing the water temperature is

$$C_w \dot{T}_w = w_w c_{vw}(T_{wi} - T_w) + hA(T_s - T_w), \tag{3}$$

where

w_w = mass flow rate of the water,
c_{vw} = specific heat of water,
T_{wi} = temperature of the incoming water,
T_w = temperature of the outflowing water.

Example 2 – Modeling of a Hydraulic Piston

Determine the differential equation describing the motion of the piston actuator shown in Figure 8, given that there is a force F_D acting on it and a pressure ‘p’ in the chamber.

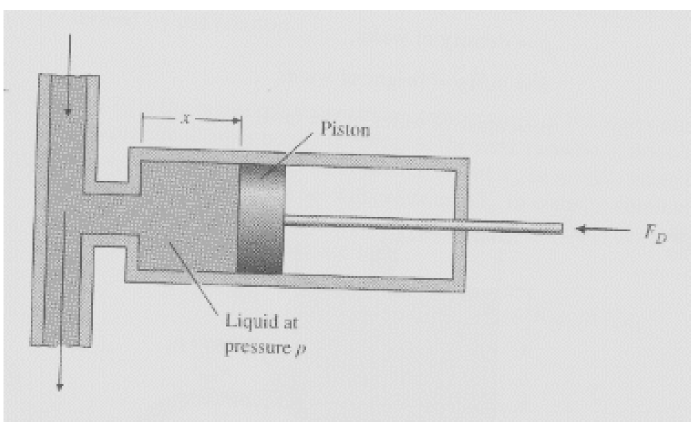

Figure 8: Hydraulic piston actuator

Writing the Newton’s second law of motion for the piston, we have

$$M \ddot{x} = Ap - F_D,$$

where

A = area of the piston,
P = pressure in the chamber,
M = mass of the piston,
X = position of the piston.

In many cases of fluid-flow problems the flow is restricted either by a constriction in the path or by friction. The general form of the effect of resistance is given by

$$w = \frac{1}{R}\left(p_1 - p_2\right)^{1/\alpha},$$

w = mass flow rate,
p_1, p_2 = pressures at ends of the path through which flow is occurring,
R, α = constants whose values depend on the type of restriction.

The constant α takes on values between 1 and 2. The most common value is approximately 2 for high flow rates through pipes or through short constrictions or nozzles. Note that for this value the flow is proportional to the square root of the pressure difference and therefore will produce a non-linear differential equation.

Reading Assignment

See separate file on textbook reading assignments depending on the text edition you own. Read examples Handout E.9 posted on the course web page.