Lab 4 – DC Servomotor Modeling

Introduction

Electric motors come in many sizes and types, but their basic function is to convert electrical energy into mechanical energy. They can be found in VCR's, elevators, CD players, toys, robots, watches, automobiles, subway trains, fans, space ships, air conditioners, refrigerators, and many other places.

D.C. motors are motors that run on Direct Current from a battery or D.C. power supply. Direct Current is the term used to describe electricity at a constant voltage. A.C. motors run on Alternating Current, which oscillates with a fixed frequency between a positive and negative value. Electrical outlets provide A.C. power. When a battery or D.C. power supply is connected between a D.C. motor's electrical leads, the motor converts electrical energy to mechanical work as the output shaft turns.

Objective

The objective of this lab is to model a system comprised of a DC motor and an inertia, and to calibrate the sensors used to measure the system states. The calibration data will then be used in subsequent labs to implement DC motor position and speed control.

Theory

The system is made up of a DC motor coupled to a load as shown below.

A disk with a total mass moment of inertia \( J \) acts as the inertial load. The motor is equipped with a tachometer, which provides a measure of the speed of the motor, and a potentiometer for...
measuring the angular position. The motor is powered by means of a power amplifier, which amplifies the current coming into the motor, but not the voltage. The developed torque of the DC motor is proportional to the magnitude of the flux due to the field current \( i_f \) and the armature current \( i_a \). The developed torque can be expressed as

\[
T(t) = K \phi i_a \quad \text{where} \quad \phi \text{ represents the magnetic flux}
\]  

(1)

For the armature control mode, the field current is held constant and an adjustable voltage is applied to the armature. The flux produced by the field current is therefore constant. Hence the torque is only proportional to the armature current and is given by

\[
T(t) = K_f i_a
\]  

(2)

When the armature is rotating, a voltage \( e_b \) is produced that is proportional to the product of the flux and the speed, \( \omega \). Because the polarity of this voltage opposes the applied voltage \( e_a \), it is also called back emf. Since the flux is constant, the induced voltage is given by

\[
e_b = K_b \omega
\]  

(3)

Looking at the circuit diagram of the motor, we get the voltage equation of the armature circuit as

\[
L \frac{di_a}{dt} + i_a R + e_b = v_f
\]  

(4)

The current in the armature produces the required torque as per (1). The required torque depends on the load connected to the motor shaft. If the load is modeled as a moment of inertia and a damper as shown in the figure, then the torque equation can be written as

\[
J \frac{d\omega}{dt} + B \omega = T(t)
\]  

(5)

Substituting (2) into (5) and solving results in

\[
i_a = \frac{1}{K_f} \left( J \frac{d\omega}{dt} + B \omega \right)
\]  

(6)

Differentiating (6) once and using the result along with (6) in (4) produces
Equation (7) represents the response of the system to an input voltage $v_i$.

If the inductance in the system can be ignored, then (7) can be simplified as below

$$\frac{LJ}{K_t} \frac{d^2 \omega}{dt^2} + \left( \frac{LB + RJ}{K_t} \right) \frac{d\omega}{dt} + \left( \frac{RB}{K_t} + K_b \right) \omega = v_i$$

(7)

The above equation is of the form,

$$sK = v_i$$

(8)

Equation (9) is a first order equation that shows how $\omega$ varies with input voltage. $K_s$ is the numerator of the first order approximation of the system dynamics. If the angular velocity is at steady state, then $K_s$ is the relation between the DC motor input voltage and the steady state velocity. This is called the steady state gain. It may be determined from first principles, as seen by comparing in equations (8) and (9) or it may be determined experimentally. When it is determined experimentally, the correct notation is $\hat{K}_s$, which shows that this is a measurement of $K_s$.

The response of this system to a unit step voltage is shown in figure (2).
A notable feature of this response pattern is that $\omega$ reaches a steady state value, which is denoted by $\omega_{ss}$. In effect,

$$\omega_{ss} = K_s v_i$$  \hspace{1cm} (10)

since $\frac{d\omega}{dt} \rightarrow 0$. This notion of steady state behavior is important in that if one were to ignore the transient response of the system, the motor can be viewed as a purely static system whose behavior is described by (10). The motor response in Figure 2 involves a transient effect best described in terms of the time constant $\tau$. It is defined as the time it takes for the response to reach 63% of its steady state, or final value.

**Instrumentation**

A **tachometer** outputs a voltage that is proportional to the angular speed $\omega$ of the motor. This voltage varies from –10 to 10 volts. At higher speeds the voltage may be more than 10 Volts. The analog output from the tachometer can be read using the Data Acquisition board. The voltage read must be converted to speed after A/D conversion.

$$\omega = K_s V_o$$  \hspace{1cm} (11)

where $V_o$ is the voltage output by the tachometer. You will determine this value in the calibration part of the experiment.
A potentiometer outputs a voltage proportional to the angular position of the motor. It is mounted directly on the shaft of the motor. The voltage $V_{o2}$ is related to $\theta$ according to the relationship

$$\theta = K_{v}V_{o2} \quad (12)$$

A power amplifier amplifies the current needed to drive the motor since the output current of the D/A is very small. The voltage $v_i$ to the motor should be kept between ±10 Volts.

**Lab Procedure**

The lab deals with calibrating the speed and position sensors and finding out the parameters of the motor – namely the steady state gain, $K_v$, and the time constant, $\tau$, of the motor.

1. **Create a Simulink model capable of sending a constant voltage to the DC motor and reading a voltage from the onboard tachometer.**
   a) Use the Simulation Interface Toolkit and LabVIEW (see SIT Tutorial) to interface with the hardware. Allow the input voltage to be varied using a LabVIEW control.
   b) Use Analog Output channel 0 on the DAQ card to output a voltage to the motor.
   c) Use Analog Input channel 0 on the DAQ card to input the tachometer voltage.
   d) Log the tachometer voltage, $V_o$, to a separate file, and display it on the screen as the model is running.
      **Note:** $V_o$ is a floating source with a ±1 V dead zone.
   e) Use a simulation step size of 0.01 seconds (a 100 Hz sampling rate).
      **Note:** The step size can be increased (equivalent to reducing sampling rate) so the time interval displayed on the graphs/charts to be created in Lab 8 is larger (LabVIEW stores 1000 data points in memory regardless of the time step).

2. **Steady State Gain Determination and Tachometer Calibration**
   a) Determine the voltage range of operation for the DC motor. (The range should be ~1 – 9 V.)
   b) Send 5 voltages across range of operation to DC motor using analog output channel 1 on the DAQ board.
      **Note:** Set $V_{out}$ to zero after each run (before stopping the VI)
   c) Measure the steady-state angular velocity, $\omega$, using a handheld stroboscopic tachometer.
   d) Save the output voltage, $V_o$, from the onboard tachometer to a separate file for the estimation of the motor time constant.
   e) The table below can be used to record your results.
   f) Find $\hat{K}_v$, an estimate of the DC motor steady state gain.
i. Take the mean of the five values of $\frac{\omega_{ss}}{v_i}$.

ii. Find the standard deviation of the estimate, $\Delta \hat{K}_s$.

g) Determine $K_v$ – take the mean of the five values of $\frac{\omega_{ss}}{V_o}$.

3. Estimation of the motor time constant.

The motor time constant, $\tau$, is shown in the sample motor response in figure 2. The motor time constant is the time it takes for the motor to reach 63% of its steady state value, when subject to a step input.

Write a MATLAB m-file or use Excel to estimate the motor time constant.

<table>
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<tr>
<th>Run</th>
<th>$v_i$ [V]</th>
<th>$V_o$ [V]</th>
<th>$\omega_{ss}$ [rpm]</th>
<th>$K_v = \frac{\omega_{ss}}{V_o}$ [rpm/V]</th>
<th>$\hat{K}<em>s = \frac{\omega</em>{ss}}{v_i}$ [rpm/V]</th>
<th>$\tau$ [sec]</th>
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4. Potentiometer calibration.

a) Modify the Simulink model used in the tachometer calibration to read the rotary potentiometer from Analog Input channel 1. Save the Simulink model under a different file name and perform the procedure listed in the SIT Tutorial on the Simulink model.

b) Use the same step size you set in part 1.e) above.

c) Do not send a voltage to the DC motor.

d) Use a numeric indicator in LabVIEW to view the value of the potentiometer voltage. The data does not need to be logged to a text file. Rather, you will record the voltage reading displayed for each angular position of the motor.

e) Manually rotate the load until the potentiometer voltage is 0 V, this corresponds to the 0º position.

f) Rotate the shaft in 45º increments in both the positive and negative direction, and record the potentiometer voltage. Be cautious around $\theta = 180º$ because the potentiometer voltage switches from positive to negative at this point and there is a small dead region.

g) Plot the angular shaft position vs. potentiometer voltage.
h) Determine $K_x$, using equation 12.

*NOTE:  Save all data and results obtained in this lab for use with Lab 8: Implementation of DC Motor Speed Control.

**Issues to be addressed in the report:**

- System Description
- Include all the data and the calibration constants obtained from the lab
- Include plots of the speed vs. time data and report the time constants you obtain from the graphs.
- Comment on whether or not the system behaves linearly, as the model predicts.

**Things you have learned from this lab**

- Modeling electro-mechanical systems.
- Calibration of sensors.
- Evaluating time constant of first order systems.