Lab 6 – Implementation of DC Motor Speed Control

Introduction

Systems in which the output has no effect on the input quantity are called open-loop control systems. They can be represented by the following functional diagram depicted in Figure 8-1.

![Open Loop Control System](image1)

Figure 8-1 – Open Loop Control System (drawn by Christopher Cullum)

On the other hand, systems that track the input by using the output are called closed loop systems. They can be represented by the following functional diagram shown in Figure 8-2.

![Closed Loop Control System](image2)

Figure 8-2 – Closed Loop Control System (drawn by Christopher Cullum)

Open loop control systems are easy to implement, but are often unable to meet performance requirements. Closed loop systems can be designed to meet desired specifications.

Objective

1. The objective of this lab is to design and implement controllers. More specifically both open and closed loop controllers will be implemented.
2. To examine the impact of parameter uncertainty. Speed control of a DC motor will be used as an example to demonstrate this.
3. To understand the difference between simulations on a simplified model and implementation on a real system.

Prelab:

The approximate plant model for the system representing the DC Motor is given by
The values of $\tau$ and $K_s$ were determined experimentally in lab 6. Use Simulink to simulate the behavior of the plant for values of $v_i$ equal to 2 and 5 volts.

**System Description and model analysis**

The system is the same DC motor for which modeling and calibration of sensors was carried out. A schematic of the system is shown in Figure 8-3.

![DC Motor System Model](image)

The simplified model of the model of the DC motor can be represented as

$$ \tau \frac{d\omega}{dt} + \omega = K_s v_i $$  \hspace{1cm} (1)

As time $t$ goes to infinity, and if $\frac{d\omega}{dt} \to 0$ and we simply have

$$ \omega_{ss} = K_s v_i $$  \hspace{1cm} (2)

The idea of open loop control is to find an input voltage, $v_i$, that produces a desired speed, $\omega_d$, at steady state. To implement the open loop control the value of $K_s$ must be known. If $K_s$ is known, then equation (2) can be used to solve for $v_i$, given the desired speed $\omega_d$. However, only an estimate of $K_s$ is known. This estimate was found in lab 6 and is denoted $\hat{K}_s$. As this is the best available estimate of $K_s$, it will be used to determine $v_i$.

$$ v_i = \hat{K}_s^{-1} \sigma_d $$  \hspace{1cm} (3)
**Open Loop Control**

In order to implement the open loop controller you need to produce the voltage $v_i$ as shown in Figure 8-4.

![DC Motor Open Loop Controller Block Diagram](image)

If the estimate of the DC motor steady state gain is accurate ($\hat{K}_s = K_s$), then the control objective can be achieved. This can be verified as follows:

$$
ss \omega = \lim_{t \to \infty} \omega (t) = K_s v_i,
$$

$$
ss \omega = K_s (\hat{K}_s^{-1} \omega_d)
$$

$$
ss \omega = K_s (K_s^{-1} \omega_d)
$$

$$
ss \omega = \omega_d
$$

(4)

(5)

On the other hand if the estimate is not correct, as is typically the case, then $\hat{K}_s \neq K_s$ and the control objective is not achieved, meaning $\omega_s \neq \omega_d$. In general, the estimate is off from the actual value, meaning

$$
\hat{K}_s = K_s + \delta K_s
$$

(6)

The value of $\delta K_s$ is unknown, since knowing its exact value would mean knowing $K_s$. However, it is assumed to be bounded by the standard deviation of $\hat{K}_s$ measured in lab 6.

$$
|\delta K_s| \leq \Delta \hat{K}_s
$$

(7)

where $\Delta \hat{K}_s$ is known. Moreover, we do know that $\Delta \hat{K}_s$ is small compared to $K_s$.

$$
\frac{\Delta \hat{K}_s}{K_s} \ll 1
$$

(8)

clearly this implies that $|\delta K_s|$ is small compared to either $\hat{K}_s$ or $K_s$. 

With the above in mind, substituting for $v_i$, the control voltage, and making note of (6) we have,

\[
\omega_{ss} = \lim_{t \to \infty} \omega(t) = K_v v_i
\]
\[
\omega_{ss} = K_v \left( \hat{K}_s^{-1} \omega_d \right)
\]
\[
\omega_{ss} = K_v \left( (K_s + \delta K_s)^{-1} \omega_d \right)
\]
\[
\omega_{ss} = \frac{K_v}{K_s + \delta K_s} \omega_d
\]
\[
\omega_{ss} = \frac{1}{1 + \frac{\delta K_s}{K_s}} \omega_d
\]

Next we make use of the identity,

\[
\frac{1}{1 + x} \approx 1 - x, \text{ for small values of } x,
\]

Rewriting (9), we get

\[
\omega_{ss} = \left( 1 - \frac{\delta K_s}{K_s} \right) \omega_d
\]
\[
\omega_{ss} = \omega_d - \frac{\delta K_s}{K_s} \omega_d
\]

This shows that we are not able to reach $\omega_d$ at steady state. Thus $\omega_{ss} \neq \omega_d$. In fact, there will be a relative error as large as $\frac{\delta K_s}{K_s}$, as is evident from (12). In effect, $\omega$ will be in the range of

\[
\omega_d - \frac{\delta K_s}{K_s} \omega_d \leq \omega \leq \omega_d + \frac{\delta K_s}{K_s} \omega_d
\]

This error can be minimized through the use of feedback control. Feedback also has the advantage of reducing the effects of disturbances acting on the system.

**Closed Loop Control**

To deal with the inaccuracies of the system, feedback is often employed. Feedback can be broken into two parts. The first part includes the use of a sensor to supply information about the output of the system. This information is then used to change the input to the system to make the output of the system equal to the desired value. This feedback of information is also known as closing the loop. A schematic of the feedforward-feedback control scheme used in this lab is shown in Figure 8-5.
With the new control input, \( v_i = \hat{k}_s^{-1} \omega_d + K_p (\omega_d - \omega) \), we have

\[
\omega_{ss} = \lim_{t \to \infty} \omega(t) = K_v v_i
\]

\[
\omega_{ss} = K_s \left( \hat{k}_s^{-1} \omega_d + K_p (\omega_d - \omega_{ss}) \right)
\]

\[
\omega_{ss} = K_s \left( K_s + \delta K_s \right)^{-1} \omega_d + K_p (\omega_d - \omega_{ss})
\]

\[
\omega_{ss} = K_s \left( \frac{1}{K_s + \delta K_s} \omega_d + K_p (\omega_d - \omega_{ss}) \right)
\]

Once again making the approximation, we have

\[
\omega_{ss} = \left( 1 - \frac{\delta K_s}{K_s} \right) \omega_d + K_s K_p (\omega_d - \omega_{ss})
\]

(15)

Now separating the terms and factoring, we have

\[
(1 + K_s K_p) \omega_{ss} = (1 + K_s K_p) \omega_d - \left( \frac{\delta K_s}{K_s} \right) \omega_d
\]

\[
\omega_{ss} = \omega_d - \frac{\delta K_s}{1 + K_s K_p} \omega_d
\]

(16)

Comparing this with expression (12), it is clear that feedback reduces the steady state error for this system. The larger the control gain, \( K_p \), the better the performance. You will verify this in the lab.
**Lab Procedure**

1. **Open Loop Controller Simulation**
   a. Use Simulink to simulate the open loop control of the motor as shown in Figure 8-4.
   b. Select six value of $\omega_d$ equally spaced across the operating range of the motor as determined in lab 6 (~100-900 rpm).
   c. To make the simulation realistic, choose $K_s$ to be different than $\hat{K}_s$ by about 10 percent.
   d. Save the system response for each run to the MATLAB workspace.
   e. Record the results in the table below.

<table>
<thead>
<tr>
<th>Run</th>
<th>$\omega_d$ [rpm]</th>
<th>$\omega_{ss}$ [rpm]</th>
<th>$\omega_d - \omega_{ss}$ [rpm]</th>
<th>$\left( \frac{\Delta K_s}{K_s} \right) \omega_d$ [rpm]</th>
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2. **Open Loop Controller Implementation**
   a. Use Simulink to implement the open loop control of the motor as shown in Figure 8-4. Use the same hardware setup and simulation settings as used in lab 6. If the time step you set in Lab 6 does not work properly in LabVIEW, then use 0.01s.
   b. Send the six values of $\omega_d$ used in the open loop controller simulation to the DC motor.
   c. Measure the steady state speed with the handheld tachometer.
   d. Save the system response in a separate file.
   e. Record the results in the table below.

<table>
<thead>
<tr>
<th>Run</th>
<th>$\omega_d$ [rpm]</th>
<th>$\omega_{ss}$ Onboard Tach [rpm]</th>
<th>$\omega_{ss}$ Handheld Tach [rpm]</th>
<th>$\omega_d - \omega_{ss}$ [rpm]</th>
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3. **Closed Loop Controller Implementation**
   a. Use Simulink to implement the closed loop control of the motor as shown in Figure 8-5. Use the same hardware setup and simulation settings as used in lab 6.
   b. Select two values of $\omega_d$ tested with the open loop controller. (It is recommend that a value at the lower end of the speed range and one at the higher end of the speed range be selected.)
   c. Select three values for the proportional control gain, $K_p$. Select values as specified by the instructor. If none are specified, use:
      i. $K_p \approx 0.01$
      ii. $0.05 \leq K_p \leq 0.45$ — find the value that yields the most desirable response
      iii. $K_p \geq 0.5$
   d. Run the experiment at both $\omega_d$ values for each $K_p$ value.
   e. Measure the steady state speed with the handheld tachometer.
   f. Save the system response in a separate file.
   g. Record the results in the table below.

<table>
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<tr>
<th>Run</th>
<th>$\omega_d$ [rpm]</th>
<th>$K_p$ [V/rpm]</th>
<th>$\omega_{ss}$ Onboard Tach [rpm]</th>
<th>$\omega_{ss}$ Handheld Tach [rpm]</th>
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4. **Closed Loop Controller Simulation**
   a. Use Simulink to simulate the closed loop control of the motor as shown in Figure 8-5.
   b. Run the simulation for the same six $\omega_d$, $K_p$ combinations run on the actual system.
   c. To make the simulation realistic, choose $K_s$ to be different than $\hat{K_s}$ by about 10 percent.
   d. Save the system response for each run to the MATLAB workspace.
   e. Record the results in the table below.
Issues to be addressed in the report:

1. Can you achieve open loop speed control with zero steady state error? Why?
2. What is the expected range of the speed of the motor for open loop control? Are the results in accordance with theory?
3. What is the effect of increasing $K_p$ on the steady state error? Why?
4. Compare the simulations of the closed loop system with the actual measurements.
5. Plot the response curve of the system for each trial. It is recommended that you plot multiple response curves on the same axes to facilitate analysis.

Things you have learned in this lab

1. Effect of parameter uncertainty
2. Open loop DC motor speed control.
3. Closed loop DC motor speed control.