HANDOUT E.17 - EXAMPLES ON BODE PLOTS OF FIRST AND SECOND ORDER SYSTEMS

Example 1

Obtain the Bode plot of the system given by the transfer function

$$G(s) = \frac{1}{2s+1}.$$

We convert the transfer function in the following format by substituting $s = j\omega$

$$G(j\omega) = \frac{1}{2j\omega + 1}. \tag{1}$$

We call $\omega = \frac{1}{2}$, the break point. So for

$\omega \ll \frac{1}{2}$, i.e., for small values of ω

$$G(j\omega) \approx 1.$$

Therefore taking the log magnitude of the transfer function for very small values of ω, we get

$$20\log|G(j\omega)| = 20\log(1) = 0.$$

Hence we see that below the break point the magnitude curve is approximately a constant.

For,

$\omega \gg \frac{1}{2}$, i.e., for very large values of ω

$$G(j\omega) \approx \frac{1}{2j\omega}.$$

Similarly taking the log magnitude of the transfer function for very large values of ω, we have

$$20\log|G(j\omega)| = 20\log\left|\frac{1}{2j\omega}\right| = 20\log\left(\frac{1}{2\omega}\right) = 20\log(1) - 20\log(2\omega) = -20\log(2\omega).$$

So we see that, above the break point the magnitude curve is linear in nature with a slope of –20 dB per decade. The two asymptotes meet at the break point. The asymptotic bode magnitude plot is shown below.
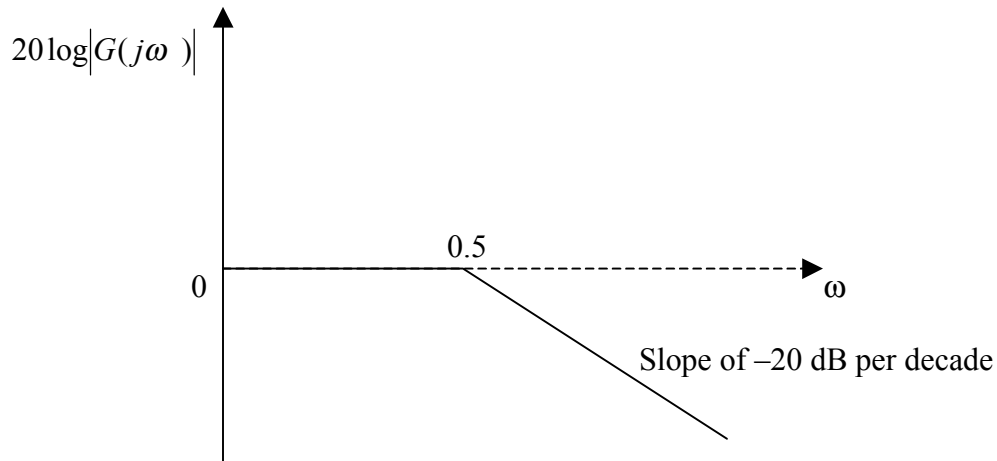
The phase of the transfer function given by equation (1) is given by

$$\phi = 0 - \tan^{-1}(2\omega) = -\tan^{-1}(2\omega).$$

So for small values of ω, i.e., $\omega \approx 0$, we get

$$\phi \approx 0.$$

For very large values of ω, i.e., $\omega \rightarrow \infty$, the phase tends to -90 degrees.

To obtain the actual curve, the magnitude is calculated at the break points and joining them with a smooth curve. The Bode plot of the above transfer function is obtained using MATLAB by following the sequence of command given.

```
num = 1;
den = [2 1];
sys = tf(num,den);
grid;
bode(sys)
```

The plot given below shows the actual curve.

Bode Diagrams
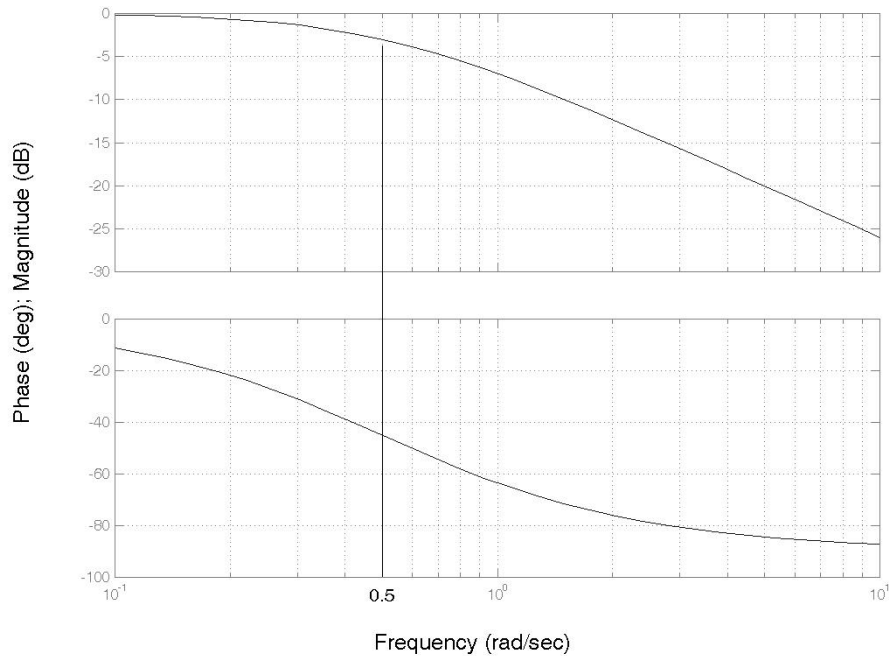
Example 2

Obtain the bode plot of the system given by the transfer function

$$G(s) = \frac{4}{s^2 + s + 4}.$$

Substituting $s = j\omega$ in the above transfer function, we get

$$G(j\omega) = \frac{4}{(j\omega)^2 + j\omega + 4}.$$

From the above transfer function, it can be concluded that $\omega_n = 2$, so therefore reducing the above transfer function by dividing both the numerator and denominator by ω_n, we get

$$G(j\omega) = \frac{1}{\left(\dfrac{j\omega}{\omega_n}\right)^2 + 2\zeta\left(\dfrac{j\omega}{\omega_n}\right) + 1} = \frac{1}{\left(\dfrac{j\omega}{2}\right)^2 + 0.5\left(\dfrac{j\omega}{2}\right) + 1}.$$

In this case the break point is $\omega = \omega_n$. Therefore for

$\omega \ll \omega_n$, i.e., for small values of ω,

$G(j\omega) \approx 1.$

Taking the log magnitude, we get
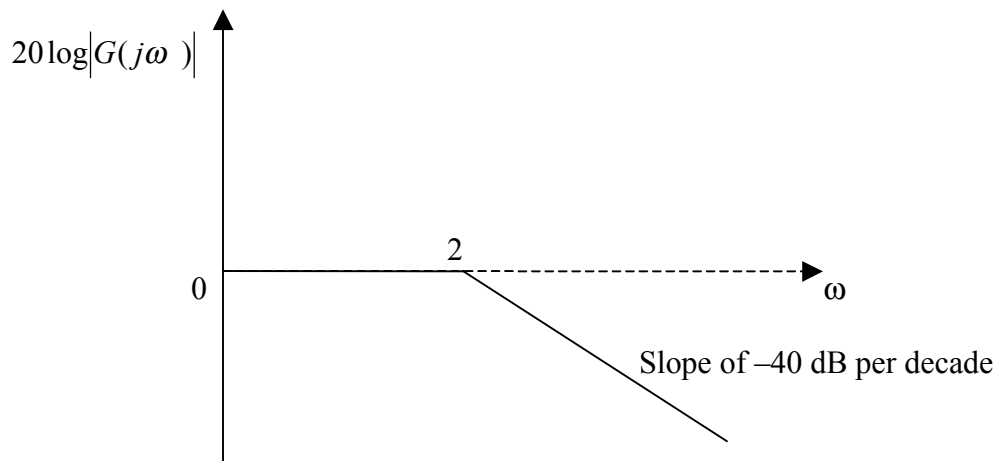
$20\log|G(j\omega)| = 20\log(1) = 0.$

Therefore the magnitude is approximately a constant below the break point. For larger values of ω i.e., for $\omega \gg \omega_n$, we get

$$G(j\omega) \approx \frac{1}{\left(\dfrac{j\omega}{\omega_n}\right)^2} = \frac{1}{\left(\dfrac{j\omega}{2}\right)^2}.$$

Taking the log magnitude, we get

$$20\log|G(j\omega)| = 20\log\left(\frac{1}{\left(\dfrac{\omega}{\omega_n}\right)^2}\right) = 20\log(1) - 20\log\left(\frac{\omega}{\omega_n}\right)^2 = -40\log\frac{\omega}{\omega_n} = -40\log\frac{\omega}{2}.$$

From the above relation, it can be concluded that the magnitude plot is linear in nature with a slope of –40 dB per decade. The asymptotic plot is as shown below.

The transfer function can be rewritten as

$G(s) = \dfrac{1}{(s+a)(s+b)},$ where 'a' and 'b' are the roots of the denominator.

Substituting $s = j\omega$, we get

$$G(j\omega) = \frac{1}{(j\omega + a)(j\omega + b)}.$$

The phase of the above transfer function is given as

$$\phi = 0 - \tan^{-1}(\frac{\omega}{a}) - \tan^{-1}(\frac{\omega}{b}).$$

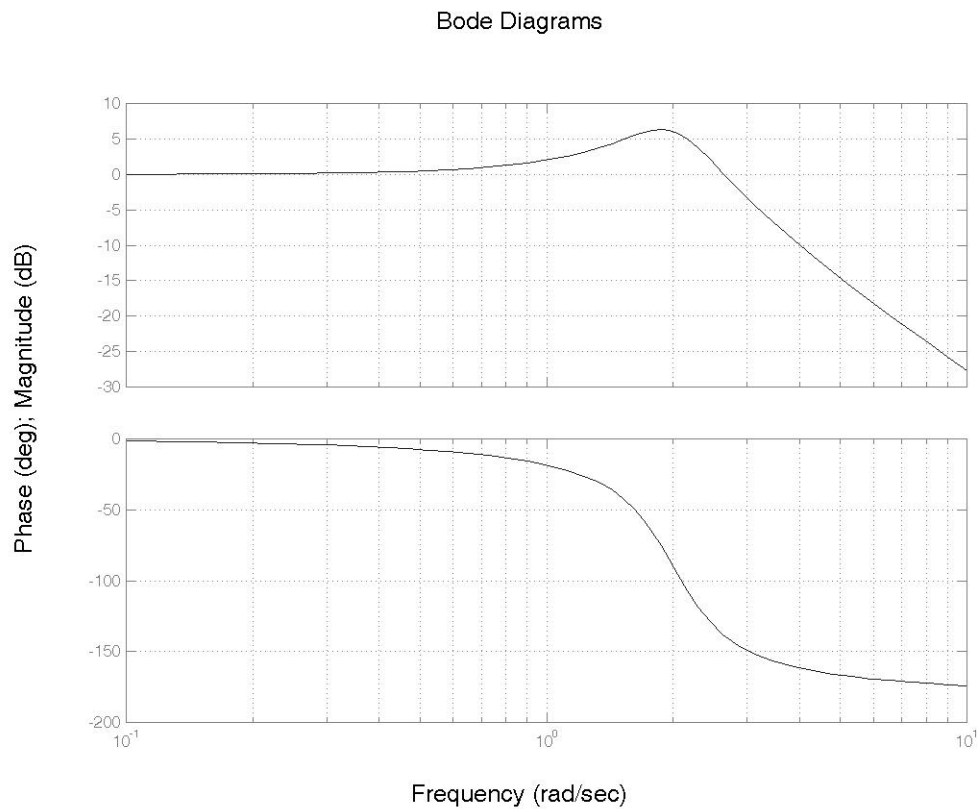So therefore for $\omega \approx 0$, we get

$$\phi \approx 0.$$

For very large values of ω, i.e., $\omega \to \infty$, the phase tends to -180 degrees.

The actual bode plot is obtained by following the given MATLAB sequence.

```
num = 4;
den = [1 1 4];
sys = tf(num,den);
grid
bode(sys)
```

The plot is attached below.

Bode Diagrams

Example 3

Plot the Bode magnitude and phase for the system with transfer function

$$G(s) = \frac{2000(s+0.5)}{s(s+10)(s+50)}.$$

Step 1: We convert the function to the form given below by substituting $s = j\omega$

$$G(j\omega) = \frac{2000(j\omega + 0.5)}{j\omega\,(j\omega + 10)(j\omega + 50)} = \frac{2\left(\dfrac{j\omega}{0.5}+1\right)}{j\omega\left(\dfrac{j\omega}{10}+1\right)\left(\dfrac{j\omega}{50}+1\right)}.$$

Step 2: We note that the term in jω is first order and in the denominator, so n = -1. Therefore, the low frequency asymptote is defined by the first term:

$$G(j\omega) = \frac{2}{j\omega}.$$

This asymptote is valid for ω < 0.1 because the lowest break point is at ω = 0.5. The magnitude plot of this term has a slope of –1 or –20 dB per decade. We locate the magnitude by passing through the value 2 at ω = 1 even though the composite curve will not go through this point because of the break point at ω = 0.5.
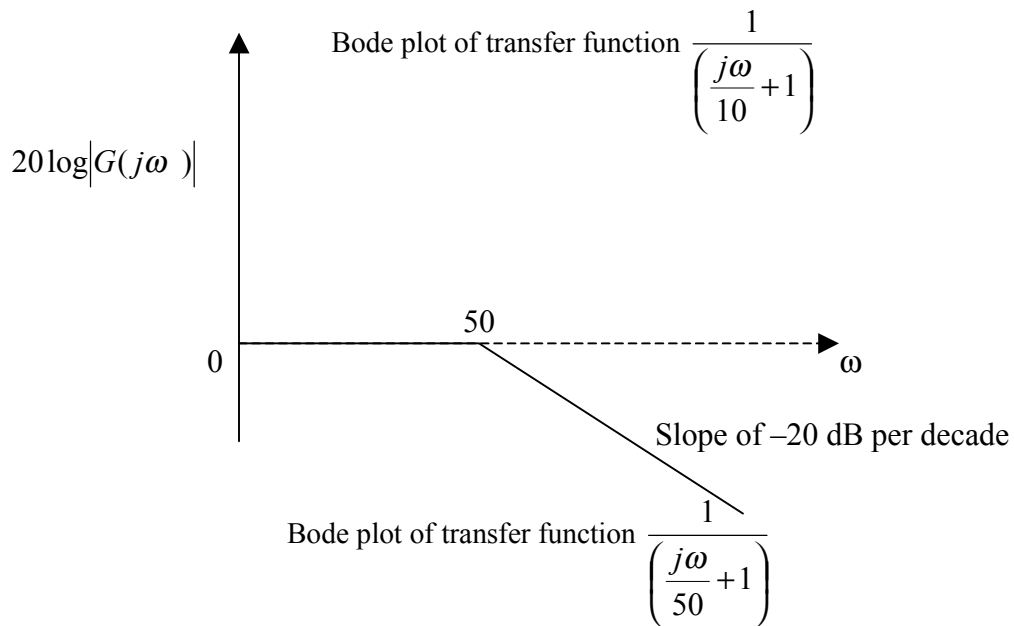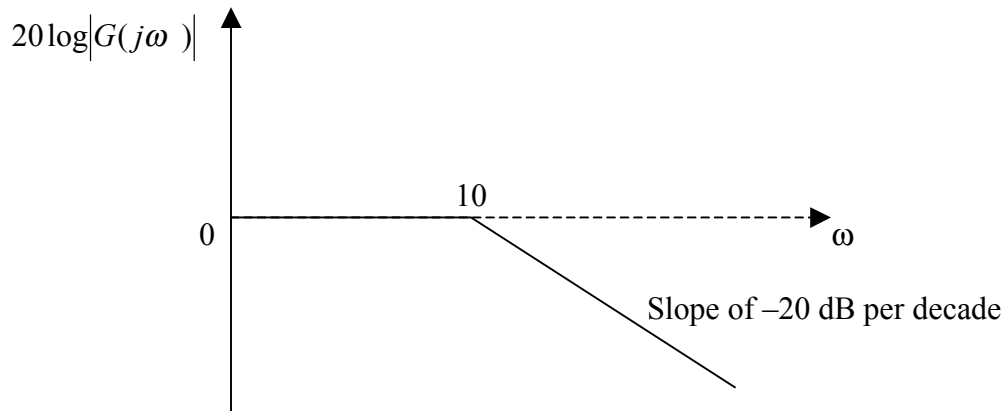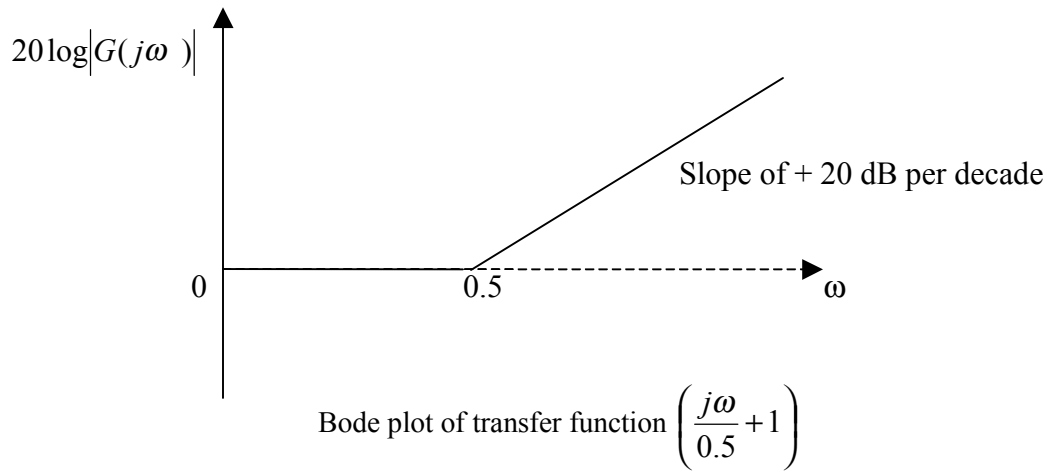
Step 3: We obtain the remainder of asymptotes as shown in the figure. First we draw a line with 0 slope that intersects the original –1 slope at ω = 0.5. Then we draw a –1 slope line that intersects the previous one at ω = 10. Finally, we draw a –2 slope line that intersects the previous –1 slope at ω = 50.
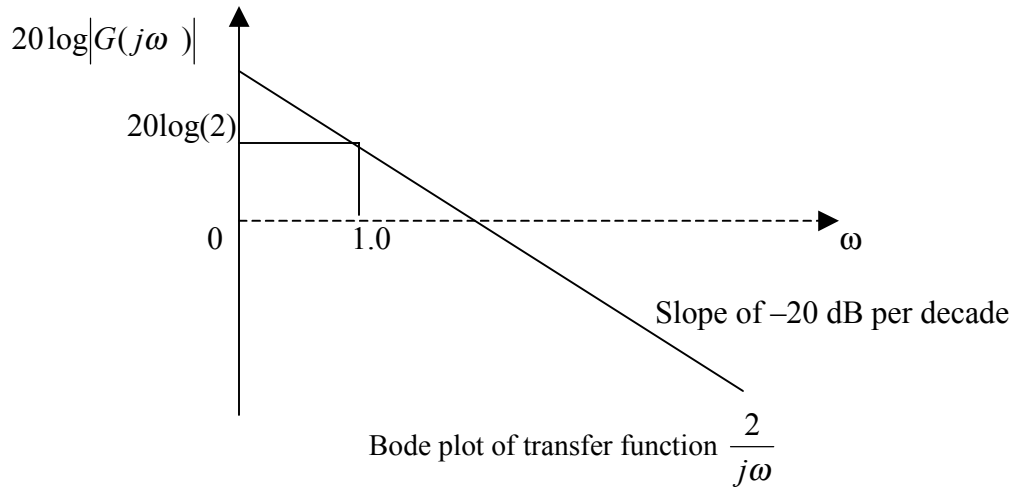
Step 4: We then sketch the actual curve by calculating the value of the magnitude at the break points and joining those points by a smooth curve. We see that the actual curve is approximately tangential to the asymptotes when far away from the break points and are a factor of 1.4 (+ 3 dB) above the asymptote at ω = 0.5 break point and a factor of 0.7 (-3 dB) below the asymptote at ω = 10 and ω = 50 break points.

Step 5: Since the phase of is $\dfrac{2}{j\omega}$ -90°, the phase curve starts at -90° at the lowest frequencies.
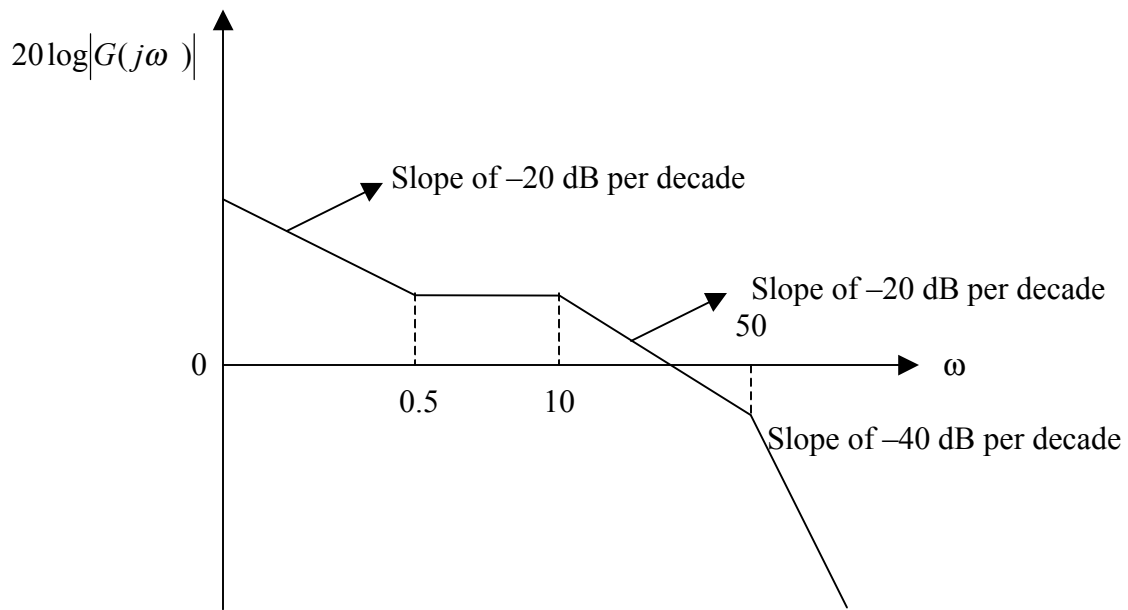
Step 6: The individual phase curves are shown in the form of dashed line. Note that the composite curve approaches each individual term.

The following plots depict the bode magnitude plot of the individual terms in the transfer function.


Bode plot of transfer function $\left(\dfrac{j\omega}{0.5}+1\right)$

Bode plot of transfer function $\dfrac{1}{\left(\dfrac{j\omega}{10}+1\right)}$

Bode plot of transfer function $\dfrac{1}{\left(\dfrac{j\omega}{50}+1\right)}$

$20\log|G(j\omega)|$

$20\log(2)$

0   1.0 ω

Slope of -20 dB per decade

Bode plot of transfer function $\dfrac{2}{j\omega}$

Combining the above bode diagrams, the composite asymptotic curve is as shown below.

$20\log|G(j\omega)|$

Slope of -20 dB per decade

Slope of -20 dB per decade
50

0

0.5   10 ω

Slope of -40 dB per decade

The actual bode magnitude curve is obtained by evaluating the actual magnitude at the break points and joining these points with a smooth curve. The actual bode plot is shown below. Similar procedure is adopted to plot the phase curve.

Bode Diagrams
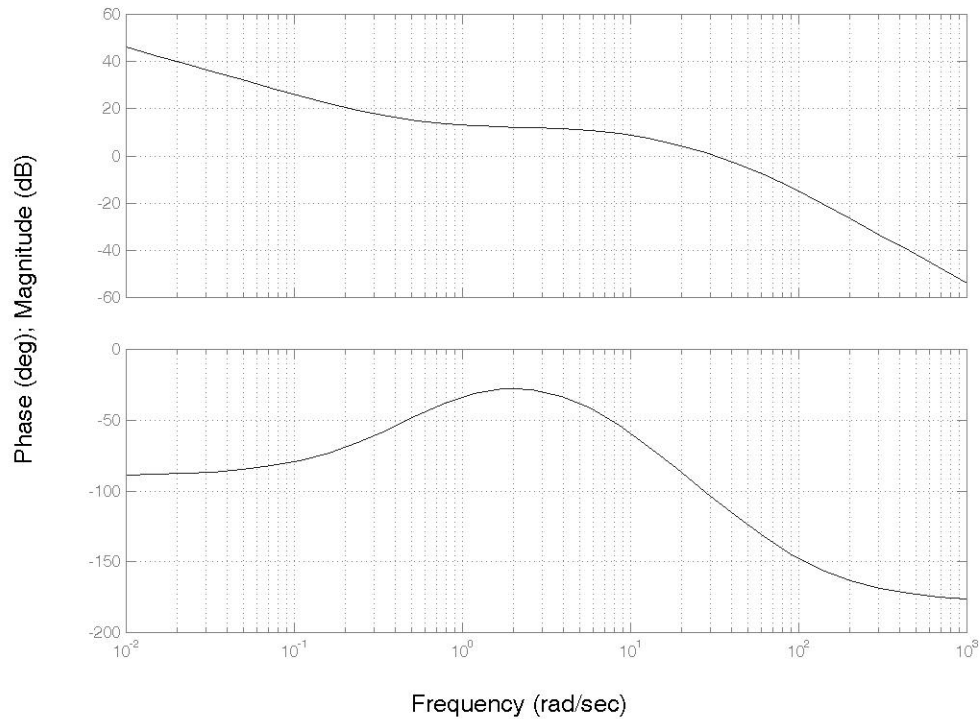
Example 4

Draw the frequency response of the system given by the transfer function
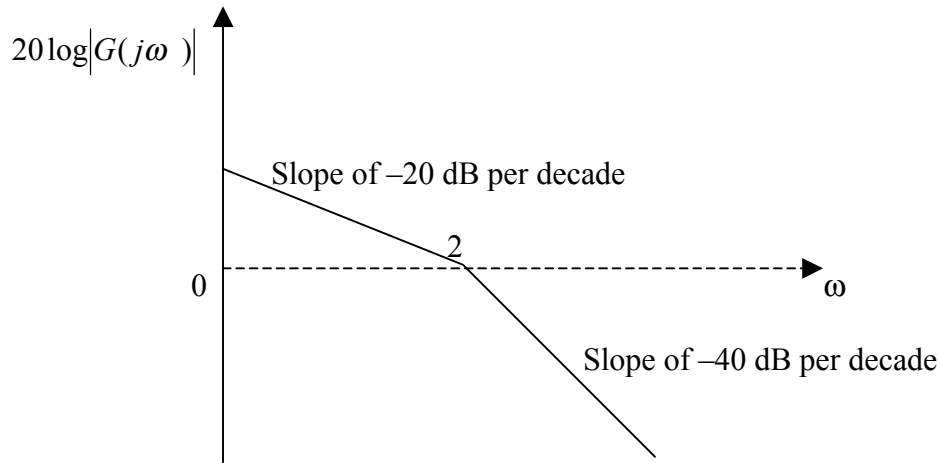
$$G(s) = \frac{10}{s(s^2 + 0.4s + 4)}.$$

Rewrite the above transfer function as

$$G(s) = \frac{10}{4} \frac{1}{s\left(\frac{s^2}{4} + 0.2\frac{s}{2} + 1\right)}.$$

Substituting $s = j\omega$ in the above transfer function, we get

$$G(j\omega) = \frac{10}{4} \frac{1}{j\omega\left(\left(\frac{j\omega}{2}\right)^2 + 0.2\frac{j\omega}{2} + 1\right)}.$$

The breakpoint for the above transfer function is at $\omega = 2$. Following the same procedure as in example 3, the composite asymptotic bode magnitude curve is as shown below.

The actual bode magnitude plot is obtained by evaluating the magnitude of the transfer function at the break point and joining it with a smooth curve. The similar procedure is adopted to obtain the phase curve for the system. The composite actual curve is as shown below.
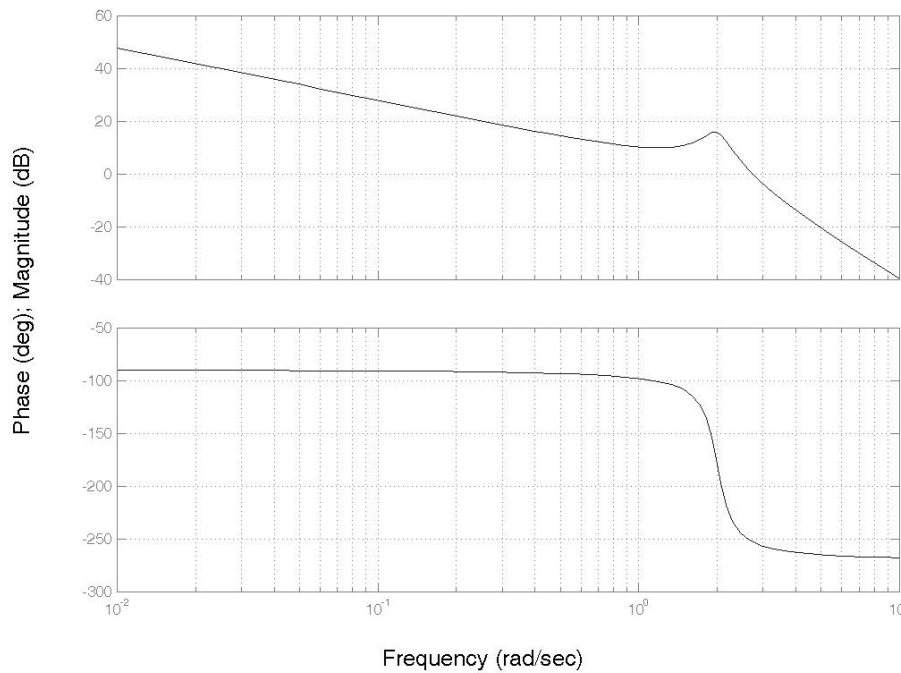
Assignment

Draw the bode plot for each of the following systems. Compare your sketches with the plots obtained using the 'bode' command in MATLAB.

a) $G(s) = \dfrac{4000}{s(s+40)}$,

b) $G(s) = \dfrac{1}{(s+1)(s^2+s+4)}$,

c) $G(s) = \dfrac{10(s+4)}{s(s+1)(s^2+25+5)}$.

Recommended Reading

"Feedback Control of Dynamic Systems" 4th Edition, by Gene F. Franklin et.al – pp 366 - 389.

Recommended Assignment

"Feedback Control of Dynamic Systems" 4th Edition, by Gene F. Franklin et.al –problem 6.3.