Robust Control for Networked Control Systems with Admissible Parameter Uncertainties

Kun Ji and Won-jong Kim*

Abstract: This paper discusses Robust $H\infty$ control problems for networked control systems (NCSs) with time delays and subject to norm-bounded parameter uncertainties. Based on a new discrete-time model, two approaches of robust controller design are proposed. A numerical example and experimental verification with an NCS test bed are given to illustrate the feasibility and effectiveness of proposed design methodologies.

Keywords: Linear matrix inequalities (LMIs), networked control system, uncertainty, time delays.

1. INTRODUCTION

Ever-increasing computational capabilities and bandwidths in the networking technology enabled researchers to develop NCSs [1-4]. Stability regions of NCSs were proposed using a hybrid-system technique by Zhang *et al*. [2]. Ji and Kim [4] proposed a real-time co-design methodology of NCSs via the Ethernet. However, controller design and the effect of the controller to the stability of the NCS were not dealt with in these papers.

A technique to stabilize delay systems was proposed by Artstein [5] by transforming a delay system into a linear finite-dimensional system. The standard H_∞ control problem for linear systems with delay was solved based on Riccati equations in [6-8]. Boyd *et al*. [9] emphasized that many problems arising in system theory could be cast into the form of LMIs. Niculescu [10] introduced an approach based on LMIs to derive sufficient conditions for the stabilization of systems with uncertain input delay. Shi *et al.* [11] discussed the problem of control of discrete time-delay linear systems with Markovian jump parameters and the time-delay case considered is state delay which is not the case of an NCS. There are some discussions about robust stability for time-delay system with parameter uncertainties and the situation of delay in the state was considered [12,13]. Similar

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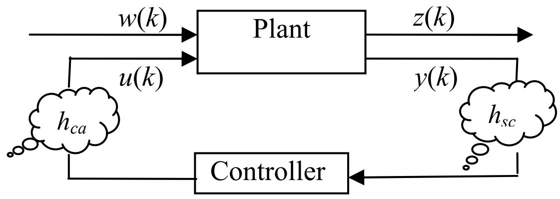

Fig. 1. An NCS with network-induced time delays.

approaches were presented in [14,15] where they focused on continuous-time models with constant time delays. In this paper, we consider an NCS framework as shown in Fig. 1.

2. PROBLEM STATEMENT

The NCS shown in Fig. 1 can be described by

$$\begin{aligned} x(k+1) &= (A+A_u)x(k) + B_w w(k) \\ &\quad + (B+B_u)u(k-h_k), \\ x(m) &= \theta(m), \forall m \in [-h_0 \quad 0), \\ z(k) &= Ex(k), \end{aligned} \tag{1}$$

where $x(k) \in R^n$ is the state, $u(k) \in R^m$ is the control input, $w(k) \in R^p$ is the disturbance input, h_k is an integer that denotes the number of the sampling periods as the length of time delay at time instant k, and $h_k = h_{sc} + h_{ca}$ for a fixed control law [2]. $\theta(\cdot)$ is the initial condition, $z(k)$ is the controlled output, A, B, B_w, and E are known real constant matrices of appropriate dimensions, and A_u and B_u are parameter uncertainties. The sampling period is T.

2.1. Assumptions

1. The admissible uncertainties are assumed to be

$$A_u = L_A F_A(k) E_A, \quad B_u = L_B F_B(k) E_B,$$

$$\bar{A}(k) = A + L_A F_A(k) E_A, \bar{B}(k) = B + L_B F_B(k) E_B, \quad (2)$$

where $F_A(k)$ and $F_B(k)$ are unknown matrices with Lebesgue measurable elements satisfying

$$\|F_A(k)\| \le 1; \quad \|F_B(k)\| \le 1, \ \forall k, \quad (3)$$

and L_A, L_B, E_A, and E_B are known real constant matrices.

2. The upper bound of the network-induced time delays is H sampling periods.
3. There exists a real scalar $S > 0$ such that for any real symmetric positive-definite matrix P and any $x(k)$, the following inequality holds.

$$S + x(k)^T Px(k) - [x(k) - x(k+1)]^T P[x(k) - x(k+1)] > 0 \quad (4)$$

2.2. Robust control problems

1. Robust Stabilization: Given $H > 0$, find a linear state-feedback control law for the system (1-3) such that the resulting closed-loop system is robustly stable for any time delay h_k satisfying $h_k \le H$ for all k.
2. Robust H_∞ Control: Given scalars $\gamma > 0$ and $H > 0$, find a linear state-feedback control law for the system (1-3) such that the resulting closed-loop system is robustly stable with disturbance attenuation γ for any time delay h_k satisfying $h_k \le H$ for all k, i.e., the closed-loop system satisfies the inequality,

$$\|T_{zw}\|_\infty \le \gamma, \quad (5)$$

where $\|T_{zw}\|_\infty$ is the H_∞ norm of the transfer function T_{zw} from $w(k)$ to $z(k)$.

3. Robust Parameter Optimization: (a) given γ, find the largest H, i.e. determine an upper bound for the time delay such that the system (1-3) is robustly stabilizable, and (b) given H, find the smallest γ, i.e. determine the lower bound of disturbance attenuation for the uncertain system (1-3).

3. ROBUST CONTROLLER DESIGN AND SUFFICIENT CONDITIONS FOR ROBUST STABILITY AND H_∞ CONTROL

Lemma 1: For any vectors $u, v \in R^n$ and any real symmetric positive-definite matrix $P \in R^{n\times n}$, the following inequality holds $-2u^T v \le u^T P^{-1} u + v^T P v$.

Lemma 2 [16]**:** Let A, L, E, and F be real matrices of appropriate dimensions with $\|F\| \le 1$. Then, the following inequalities hold.

(1) For any real symmetric positive-definite matrix P and scalar $s > 0$ such that $sI - EPE^T > 0$,

$$(A + LFE)P(A + LFE)^T \le APA^T + APE^T (sI - EPE^T)^{-1} EPA^T + sLL^T.$$

(2) For any real symmetric positive-definite matrix P and scalar $s > 0$ such that $P - sLL^T > 0$,

$$(A + LFE)^T P^{-1}(A + LFE) \le A^T (P - sLL^T)^{-1} A + s^{-1}E^T E.$$

(3) For any scalar $s > 0$, $LFE + E^T F^T L^T \le s^{-1}LL^T + sE^T E$.

3.1. Memoryless controller design

We design memoryless controller (6) to solve robust-control problems 1, 2, and 3 for system (1-3),

$$u(k) = Kx(k), \quad (6)$$

where $K \in R^{m\times n}$ is a constant matrix.

3.1.1 Robust stabilization problem

Assuming $w(k) = 0$, we present the following two theorems for the uncertain system (1-3).

Theorem 1: Given $H > 0$, the system (1-3) is robustly closed-loop stable with a control input of (6) for any time delay h_k satisfying $h_k \le H$ if there exist real symmetric positive-definite matrices P, P_1, P_2, and Q, a matrix K, and scalars $r_i > 0$, $i = 1, \ldots, 5$, such that following inequalities hold

$$R(P, P_1, P_2, Q, K, H, r_i) < 0, \quad (7)$$

$$r_3 I - E_B Q E_B{}^T > 0, \ P_1 - r_4 L_A L_A{}^T > 0, \ P_2 - r_5 L_B L_B{}^T > 0, \quad (8)$$

$$P^{-1} - P_1 - P_2 \ge 0, \ Q - KP^{-1}K^T \ge 0, \quad (9)$$

where

$$\begin{aligned} &R(P,\ P_1,\ P_2,\ Q,\ K,\ H,\ r_i) \\ &\equiv (A + BK)^T P + P(A + BK) + P(r_1 L_A L_A{}^T + r_2 L_B L_B{}^T)P \\ &+ r_1^{-1} E_A{}^T E_A + r_2^{-1} K^T E_B{}^T E_B K \\ &+ HP\Big[BQB^T + r_3 L_B L_B{}^T \\ &\qquad + BQE_B{}^T (r_3 I - E_B Q E_B{}^T)^{-1} E_B Q B^T\Big]P \\ &+ H\Big[(A - I)^T (P_1 - r_4 L_A L_A{}^T)^{-1}(A - I) + r_4^{-1} E_A{}^T E_A \\ &+ K^T B^T (P_2 - r_5 L_B L_B{}^T)^{-1} BK + r_5^{-1} K^T E_B{}^T E_B K\Big]. \end{aligned} \quad (10)$$

Proof: From (1-2) and (6), we obtain

$$x(k+1) - x(k) = (\bar{A}(k) - I)x(k) + \bar{B}(k)Kx(k - h_k),$$

$$x(k-h_k)=x(k)-\sum_{i=-h_k}^{-1}\Big[(\bar{A}(k+i)-I)x(k+i)+\bar{B}(k+i)Kx(k-h_k+i)\Big],$$
$$x(k+1)-x(k)=(\bar{A}(k)+\bar{B}(k)K-I)x(k)-\bar{B}(k)K\sum_{i=-h_k}^{-1}\Big[(\bar{A}(k+i)-I)x(k+i)+\bar{B}(k+i)Kx(k-h_k+i)\Big]. \tag{11}$$

Define a discrete-time Lyapunov function as

$$V(x,k)=x^T(k)Px(k)+W(x,k)+S(x,k), \tag{12}$$

where P is a real symmetric positive-definite matrix,

$$W(x,k)\equiv\sum_{j=-h_k}^{-1}\sum_{i=j}^{-1}x^T(k+i)\Big[\bar{A}(k+i)-I\Big]^T P_1^{-1}\Big[\bar{A}(k+i)-I\Big]x(k+i)+\sum_{j=-h_k}^{-1}\sum_{i=j-h_k}^{-1}x^T(k+i)\Big[K^T\bar{B}^T(k+i+h_k)P_2^{-1}\bar{B}(k+i+h_k)K\Big]x(k+i), \tag{13}$$

$$S(x,k)\equiv S(x,0)+\sum_{i=-k}^{-1}x^T(k+i)Px(k+i)-\sum_{i=-k}^{-1}[x(k+i)-x(k+i+1)]^T P[x(k+i)-x(k+i+1)], \tag{14}$$

where P_1 and P_2 are real symmetric positive-definite matrices to be chosen.

From (12), we obtain

$$\Delta V(x,k)=V(x,k+1)-V(x,k)=\Delta X+\Delta W+\Delta S, \tag{15}$$

$$\Delta X=x^T(k+1)Px(k+1)-x^T(k)Px(k),\quad \Delta W=W(x,k+1)-W(x,k),\quad \Delta S=S(x,k+1)-S(x,k). \tag{16}$$

For ΔX, we have

$$\Delta X=x^T(k)P\big[x(k+1)-x(k)\big]+\big[x(k+1)-x(k)\big]^T Px(k)+\Delta X_1, \tag{17}$$

where $\Delta X_1=\big[x(k+1)-x(k)\big]^T P\big[x(k+1)-x(k)\big]$.

Substituting (11) into (17), and applying Lemmas 1 and 2, we obtain

$$\begin{aligned}\Delta X\le x^T(k)\Big\{&(A+BK-I)^T P+P(A+BK-I)\\&+P(r_1L_AL_A^T+r_2L_BL_B^T)P\\&+r_1^{-1}E_A^TE_A+r_2^{-1}K^TE_B^TE_BK\\&+HP\Big[BQB^T+r_3L_BL_B^T\\&+BQE_B^T(r_3I-E_BQE_B^T)^{-1}EQB^T\Big]P\Big\}x(k)\\&+\Delta X_1+\Delta X_2+\Delta X_3,\end{aligned} \tag{18}$$

where $Q\ge KP^{-1}K^T$, $P^{-1}-P_1-P_2\ge 0$, and

$$\Delta X_2=\sum_{i=-h_k}^{-1}x^T(k+i)\Big[(\bar{A}(k+i)-I)^T P_1^{-1}(\bar{A}(k+i)-I)\Big]x(k+i),$$
$$\Delta X_3=\sum_{i=-h_k}^{-1}x^T(k-h_k+i)\Big[K^T\bar{B}^T(k+i)P_2^{-1}(k+i)\bar{B}(k+i)K\Big]x(k-h_k+i). \tag{19}$$

For ΔW, from (13) we obtain

$$\begin{aligned}\Delta W=&\,h_kx^T(k)\Big[(\bar{A}^T(k)-I)P_1^{-1}(\bar{A}(k)-I)\Big]x(k)\\&+h_kx^T(k)\Big[K^T\bar{B}^T(k+h_k)P_2^{-1}\bar{B}(k+h_k)K\Big]x(k)\\&-\sum_{i=-h_k}^{-1}x^T(k+i)\Big[(\bar{A}^T(k+i)-I)P_1^{-1}(\bar{A}(k+i)-I)\Big]x(k+i)\\&-\sum_{i=-h_k}^{-1}x^T(k-h_k+i)\Big[K^T\bar{B}^T(k+i)P_2^{-1}(k+i)\bar{B}(k+i)K\Big]x(k-h_k+i).\end{aligned}$$

Applying Lemma 2(2) and Assumption 2, we obtain

$$\begin{aligned}\Delta W\le x^T(k)\Big\{&H\Big[(A-I)^T(P_1-r_4L_AL_A^T)^{-1}(A-I)\\&+r_4^{-1}E_A^TE_A+K^TB^T(P_2-r_5L_BL_B^T)^{-1}BK\\&+r_5^{-1}K^TE_B^TE_BK\Big]\Big\}x(k)-\Delta X_2-\Delta X_3.\end{aligned} \tag{20}$$

For ΔS, from (14) we obtain

$$\Delta S\le x^T(k)(P+P)x(k)-\Delta X_1. \tag{21}$$

Substituting (18-21) into (16), we obtain

$$\Delta V(x,k)=\Delta X+\Delta W+\Delta S\le x^T(k)R(P,P_1,P_2,P_3,Q,K,H,r_i)x(k), \tag{22}$$

$$\Delta V(x,k)\le x^T(k)R(P,P_1,P_2,P_3,Q,K,H,r_i)x(k)<0. \tag{23}$$

The above sufficient condition is equivalent to the solvability of a system of following LMIs.

Theorem 2: Given an $H > 0$, the system (1-3) is robustly closed-loop stable through a control input of (6) for any time delay h_k satisfying $h_k \le H$ if there exist real symmetric positive-definite matrices Y, P_1, P_2, and Q, a matrix Z, and scalars $r_i > 0$, $i = 1,\dots, 5$, such that the following LMIs hold:

$$\begin{bmatrix} M_{11}(Y,Z) & M_{12}(Y,Z) & M_{13} & M_{14}(Y,Z,H) \\ M_{12}{}^{T}(Y,Z) & M_{22} & 0 & 0 \\ M_{13}{}^{T} & 0 & M_{33} & 0 \\ M_{14}{}^{T}(Y,Z,H) & 0 & 0 & M_{44} \end{bmatrix} < 0, \tag{24}$$

$$\begin{bmatrix} Q & Z \\ Z^T & Y \end{bmatrix} \ge 0, \tag{25}$$

$$Y - P_1 - P_2 \ge 0,\ Y - P_1 - P_2 \ge 0, \tag{26}$$

where

$$M_{11}(Y,Z) = YA^T + AY + Z^T B^T + BZ + HBQB^T + r_1 L_A L_A{}^T + r_2 L_B L_B{}^T + Hr_3 L_B L_B{}^T, \tag{27}$$

$$M_{12}(P,K) = \begin{bmatrix} YE_A{}^T & Z^T E_B{}^T \end{bmatrix}, \tag{28}$$

$$M_{13} = HBQE_B{}^T, \tag{29}$$

$$M_{14}(Y,Z,H) = H\begin{bmatrix} Y(A^T - I) & YE_A{}^T & Z^T B^T & Z^T E_B{}^T \end{bmatrix}, \tag{30}$$

$$M_{22} = -\operatorname{diag}\{r_1 I, r_2 I\}, \tag{31}$$

$$M_{33} = -H(r_3 I - E_B Q E_B{}^T), \tag{32}$$

$$M_{44} = -H\operatorname{diag}\left\{P_1 - r_4 L_A L_A{}^T, r_4 I, P_2 - r_5 L_B L_B{}^T, r_5 I\right\}, \tag{33}$$

and a stabilizing controller is $u(k) = ZY^{-1}x(k)$.

Proof: Define the new variables Y and Z in (24-26) as

$$Y = P^{-1}, Z = KY. \tag{34}$$

Multiplying both the sides of the inequality (23) by Y, and then by Schur complements, we obtain that the conditions in (7-9) are equivalent to the LMIs (24-26).

3.1.2 Robust H_∞ control problem

With $w(k) \neq 0$, we present a sufficient condition as the following theorem for the uncertain system (1-3) to be robustly stable with prescribed level of disturbance attenuation.

Theorem 3: Given an $H > 0$ and $\gamma > 0$, the system (1-3) is robustly closed-loop stable with disturbance attenuation γ with a control input of (6) for any time-delay h_k satisfying $h_k \le H$ if there exist real symmetric positive-definite matrices Y, P_1, P_2, P_3, and Q, a matrix Z, and scalars $r_i > 0$, $i = 1, \dots, 5$, such that the following LMIs hold:

$$\begin{bmatrix} M_{11}(Y,Z) & M_{12}(Y,Z) & M_{13} & M_{14}(Y,Z,H) & M_{15}(Y) \\ M_{12}{}^{T}(Y,Z) & M_{22} & 0 & 0 & 0 \\ M_{13}{}^{T} & 0 & M_{33} & 0 & 0 \\ M_{14}{}^{T}(Y,Z,H) & 0 & 0 & M_{44} & 0 \\ M_{15}{}^{T}(Y) & 0 & 0 & 0 & M_{55} \end{bmatrix} < 0, \tag{35}$$

$$\begin{bmatrix} Q & Z \\ Z^T & Y \end{bmatrix} \ge 0, \tag{36}$$

$$Y - P_1 - P_2 - P_3 \ge 0, \tag{37}$$

where

$$M_{15} = \begin{bmatrix} YE^T & B_w & 0 \end{bmatrix}, \tag{38}$$

$$M_{55} = -\begin{bmatrix} I & 0 & 0 \\ 0 & \gamma^2 I & HB_w{}^T \\ 0 & HB_w & HP_3 \end{bmatrix}, \tag{39}$$

and a guaranteed controller is $u(k) = ZY^{-1}x(k)$.

Proof: Apply (6) to (1-3), the closed-loop transfer function T_{zw} from $w(k)$ to $z(k)$ is given by

$$T_{zw} = E(zI - \bar{A} - \bar{B}Kz^{-h_k})^{-1} B_w. \tag{40}$$

From (5) and (40), and by the result of Theorem 2, Lemma 2, and Schur complements, we obtain Theorem 3.

3.1.3 Robust-parameter optimization

The problem of finding the largest H for a given γ or the smallest γ for a given H, can be easily solved using standard LMI approaches by Theorem 3. For instance, the largest H which ensures that the system (1-3) is robustly stabilizable with disturbance attenuation γ can be determined by solving the following quasi-convex optimization problem:

LMIs: (35-37),

Objective: Maximize H subject to

$$H > 0, Y > 0, P_1 > 0, P_2 > 0, P_3 > 0,$$
$$Q > 0, Z > 0, r_i > 0, i = 1, \dots, 5.$$

On the other hand, the smallest γ obtainable from Theorem 3 which ensures that the system (1-3) with a given H is robustly stabilizable can be determined by solving the following quasi-convex optimization problem:

LMIs: (35-37),

Objective: Minimize γ^2 subject to

$$Y>0,\ P_1>0,\ P_2>0,\ P_3>0,$$
$$Q>0,\ Z>0,\ r_i>0,\ i=1,...,5.$$

3.2. Dynamic controller design

A novel approach of using previously stored control data to compensate for time delays and packet losses is discussed in [17]. Here we give a sufficient condition.

Assuming $w(k)$ = 0, the discrete-time analogue of Artstein transform [5] for the system (1-3) is given by

$$\tilde{x}(k)=x(k)+\sum_{i=-h_k}^{-1}\bar{A}(k)^{-(h_k+i+1)}\bar{B}(k)u(k+i). \quad (41)$$

Lemma 3: Let ($x(k)$, $u(k)$) be a solution (admissible pair) for (1-3), defined by initial condition ($x(0)$, $u_0(.)$). Then $(\tilde{x}(k),u(k))$ with $\tilde{x}(k)$ defined by (41) is a solution (admissible pair) for the system

$$\tilde{x}(k+1)=\bar{A}(k)\tilde{x}(k)+\bar{A}(k)^{-h_k}\bar{B}(k)u(k) \quad (42)$$

with the initial condition $(\tilde{x}(0),u_0(.))$. Conversely, let $(\tilde{x}(k),u(k))$ be a solution of (42) defined by some initial condition $(\tilde{x}(0),u_0(.))$. Then given some $u_0(.)$ defined on $[-h_0\ \ 0)$ and taking

$$x(0)=\tilde{x}(0)-\sum_{i=-h_0}^{-1}\bar{A}(0)^{-(h_0+i+1)}\bar{B}u(i), \quad (43)$$

the solution for (1) by the initial condition (43) is given by

$$x(k)=\tilde{x}(k)-\sum_{i=-h_k}^{-1}\bar{A}(k)^{-(h_k+i+1)}\bar{B}(k)u(k+i). \quad (44)$$

Lemma 4: Let $u(k)=F\tilde{x}(k)$ be a feedback controller stabilizing the system (42). Then the controller

$$u(k)=F\left[x(k)+\sum_{i=-h_k}^{-1}\bar{A}(k)^{-(h_k+i+1)}\bar{B}(k)u(k+i)\right] \quad (45)$$

is stabilizing the system (1-3).

The proof is straightforward and relies entirely on Lemma 3. Since the admissible uncertainties are unknown, the dynamic controller is given as

$$\hat{u}(k)=F\left[x(k)+\sum_{i=-h_k}^{-1}A^{-(h_k+i+1)}B\hat{u}(k+i)\right]. \quad (46)$$

This compensator can be constructed by steps.

The corresponding nominal state equation of the transformed system is

$$\begin{aligned}\hat{x}(k+1)&=A\hat{x}(k)+A^{-h_k}B\hat{u}(k),\\ \hat{u}(k)&=F\hat{x}(k).\end{aligned} \quad (47)$$

Thus the augmented system equation is

$$\begin{bmatrix}\tilde{x}(k+1)\\ \hat{x}(k+1)\end{bmatrix}=\begin{bmatrix}\bar{A}(k) & \bar{A}(k)^{-h_k}\bar{B}F\\ 0 & A+A^{-h_k}BF\end{bmatrix}\begin{bmatrix}\tilde{x}(k)\\ \hat{x}(k)\end{bmatrix}. \quad (48)$$

Theorem 4: Given an H > 0, the system (1-3) is robustly closed-loop stable through a dynamic control input of form (46) for any time-delay h_k satisfying $h_k \le H$ if the largest singular value of $\begin{bmatrix}\bar{A}(k) & \bar{A}(k)^{-H}\bar{B}F\\ 0 & A+A^{-H}BF\end{bmatrix}$ is less than 1.

The proof is straightforward.

4. NUMERICAL EXAMPLE

Consider NCS shown in Fig. 1 with parameter uncertainties and time delay described by

$$\begin{aligned}x(k+1)&=(A+A_u)x(k)+B_w w(k)+(B+B_u)u(k-h_k),\\ z(k)&=Ex(k),\end{aligned} \quad (49)$$

where sampling period T is 0.01 s and

$$A=\begin{bmatrix}1 & 0.05\\ 0.01 & 0\end{bmatrix}, B=\begin{bmatrix}0.002\\ 0.01\end{bmatrix}, B_w=\begin{bmatrix}0.001\\ 0\end{bmatrix}, E=\begin{bmatrix}1 & 0\end{bmatrix},$$
$$L_A=\begin{bmatrix}0 & 0\\ 0.001 & 0.001\end{bmatrix}, L_B=\begin{bmatrix}0.001\\ 0\end{bmatrix}, E_A=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}, E_B=1.$$

1. Applying the robust control result of Theorem 2 to the above system (49) and solving LMIs (24-26) gives that the upper bound of H is 8.46.
2. Applying Theorem 3 to the above system (49) and solving LMIs (35-37) with H = 8.46 and γ = 3 gives the memoryless state-feedback control gain $K=[-10.1\ \ -10.4]$.
3. (a) Finding the maximum H: Given γ = 3, applying the H_∞ control result of Theorem 3 to the system (49) gives that the maximum H is 8.4. If given γ = 0.5, then H decreases to 2.6. (b) Finding the minimum γ: Given H = 1, applying the H_∞ control result of Theorem 3 to the system (49) gives that for time delay h_k satisfying $h_k \le 1$, the smallest value of γ is 0.2. If given H = 8, the smallest value of γ increases to 1.7.

The relation between the maximum H and the minimum γ is shown in Fig. 2. As expected, H monotonically increases as γ gets greater. However,

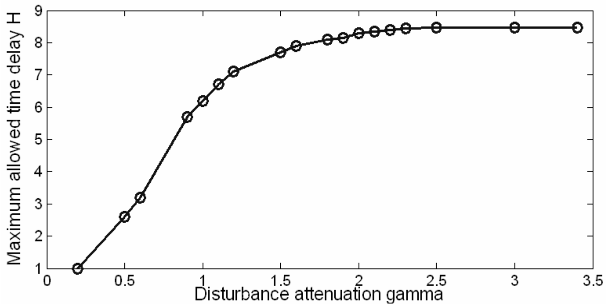

Fig. 2. Relation between the maximum H and the minimum γ.

the disturbance attenuation performance beyond a certain threshold ($\gamma \approx 2.3$) makes little difference, and H approaches 8.46.

5. EXPERIMENTAL VERIFICATION

We constructed an NCS as shown in Fig. 1. A ball magnetic-levitation (maglev) setup [17] is used. The discrete-time model of this maglev system is

$$x(k+1) = \begin{bmatrix} 2 & -0.25 \\ 4 & 0 \end{bmatrix} x(k) + \begin{bmatrix} 0.003906 \\ 0 \end{bmatrix} u(k), \quad z(k) = \begin{bmatrix} -0.00374 & -0.000935 \end{bmatrix} x(k), \tag{50}$$

where the sampling period T is 3 ms, and

$$L_A = \begin{bmatrix} 0.01 & 0.01 \\ 0 & 0 \end{bmatrix}, L_B = \begin{bmatrix} 0.0001 \\ 0 \end{bmatrix}, E_A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, E_B = 1.$$

Applying the robust control result of Theorem 2 to (50), the upper bound of H was found to be 2. The maximum round-trip time delay induced by Ethernet in out lab is about 3.4 ms which is less than 2 T, thus the maglev system is supposed to be stabilizable with a control loop closed over the Ethernet. The response of the system is shown in Fig. 3. The ball maintained its equilibrium position and did not fall down.

Then we increase the length of time delay to 3 T (9 ms) after 12 s, the system response is shown in Fig. 4. The zero value of the vertical axis denotes that the system lost its stability and that the ball could not maintain its equilibrium position and fell down.

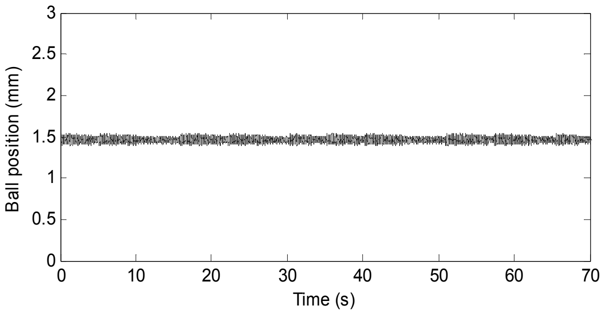

Fig. 3. NCS test bed response with a control loop closed over the Ethernet.

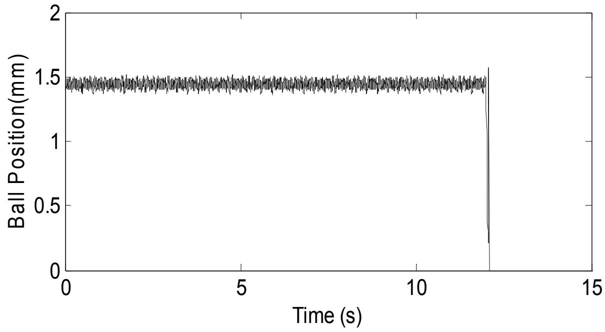

Fig. 4. NCS test bed response with 3 T-long time delay occurring after 12 s.

Thus from Figs. 3 and 4, we can conclude that the upper bound of time delay this ball maglev system can accommodate is 2 T, i.e., $H = 2$, which is consistent with that from Theorem 2.

6. CONCLUSIONS

Robust-control strategies for NCSs were proposed in this paper based on a discrete-time NCS model. Delay-dependent methods of designing linear memoryless state-feedback controllers and dynamic state-feedback controllers to solve robust control problems were presented. We provided new robust stability criteria in terms of LMIs. A numerical example was worked out to illustrate the presented robust control methodologies. We also used our NCS test bed to experimentally verify the feasibility and effectiveness of the proposed design methodologies.

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